Seat No. : _______________

MO-137

March-2019

M.Sc., Sem.-IV

510 : Mathematics
(Quantitative Techniques)
(Old Course)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (i) Derive probability of x-successes for binomial distribution. **14**

(ii) Find mean of beta distribution.

OR

(B) (i) Find even moments of normal distribution.

(ii) For the rectangular population $dF = dx$, $0 \le x \le 2$, find $\mu_1'(0)$ and μ_2.

(C) Attempt any **four** : **4**

(i) If an event takes place surely, its probability will be

(a) 1 (b) –1

(c) 0 (d) ∞

(ii) If the events A and B are independent, then P(AB)

(a) P(A) + P(B) (b) P(A) – P(B)

(c) $P(A) \cdot P(B)$ (d) P(A) / P(B)

(iii) If the events A and B are independent, the conditional probability of B given A, i.e. P(B/A) is

(a) P(A) (b) P(B)

(c) $P(A) \cdot P(B)$ (d) $P(A \cap B)$

(iv) When one should use Poisson distribution ?

(v) How one can estimate probability distribution ?

(vi) For which distribution mean and variance are same ?

2. (A) (i) Derive an EOQ model when replenishment rate is infinite and shortages are not allowed. **14**

(ii) Discuss EOQ problem with inventory level constraint.

A small shop produces three machine parts A, B and C in lots. The shop has limited storage space sufficient only for 300 units of all type of items. The relevant data for the three items is given below :

Item	**A**	**B**	**C**
Demand rate (units / month)	400	1,000	1,200
Cost per unit (₹)	5	10	15
Set-up cost per lot (₹)	150	120	180

The inventory carrying charges for the shop are 20% of the average inventory valuation per month for each item. If stock-outs are not allowed, determine the optimum lot size for each item.

OR

(B) (i) Discuss EOQ problem with investment constraint.

Consider a shop which stores three items. The demand rate for each item is constant and can be assumed to be deterministic. Shortages are not allowed. The relevant data for the items is given in the following table :

Item	**A**	**B**	**C**
Demand rate (units / year)	20	10	40
Holding cost (₹)	0.30	0.10	0.20
Set-up cost per lot (₹)	15	25	10
Purchase cost per unit (₹)	12	10	8

Determine the optimum lot size for each item when retailer has ₹ 1,500.

(ii) Derive probabilistic order level model.

(C) Attempt any **four** : **4**

(i) If small orders are placed frequently, then total inventory cost is

(a) reduced (b) increased

(c) either reduced or increased (d) minimized

(ii) Economic Order Quantity (EOQ) results in

(a) equalization of holding cost and ordering cost.

(b) minimization of set-up cost.

(c) favourable ordering cost.

(d) reduces chances of stock-outs.

(iii) If the unit purchase cost increase, the optimum order quantity

(a) increases (b) decreases

(c) either increases or decreases (d) no change

(iv) What are the types of inventory ?

(v) Define Shortage cost and explain.

(vi) Define lead-time.

3. (A) (i) Derive difference-differential equations for ((M / M / N) : (∞ / FCFS)) queue. **14**

(ii) Derive average number of customers in ((M / M / 1) : (N / FCFS)) queue.

OR

(B) (i) Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 10 per hour. The time required to serve a customer has an exponential distribution with a mean of 80 seconds. Find the waiting time of a customer.

(ii) Cars arrive at a petrol pump with exponential inter-arrival times having mean 10 minutes. The attendant takes an average of 1/4 minute per car to supply petrol, the service time being exponentially distributed. Determine (a) the average number of cars waiting to be served, (b) the average number of cars in the queue, and (c) the proportion of time for which the pump attendant is idle.

(C) Attempt any **Three** : **3**

(i) Define Ranging.

(ii) Steady and transient state.

(iii) Define different type of queue discipline.

(iv) Which distribution is followed by inter-arrival time ?

4. (A) (i) Discuss a model for the replacement of items whose maintenance costs increase with time and value of money remains same during the period. **14**

(ii) A company trading motor vehicle spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For an item, the following information is obtained :

Demand (units per day)	1	2	3	4	5
Probability	0.1	0.2	0.3	0.30	0.10

Each time an order is placed, the company incurs an ordering cost of ₹ 20 per order. The company also incurs carrying cost of ₹ 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock.

The manager of the company wishes to compare two options, for his inventory decision.

I. Order 12 units when the inventory at the beginning of the day plus order standing is less than 12 units.

II. Order 10 units when the inventory at the beginning of the day plus order standing is less than 10 units.

On first day, the company has a stock of 17 units.

The sequence of random numbers to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32, 52 using first number for day one.

Carry out the simulation for 10 days and recommend which option is advantageous to the manager.

OR

(B) (i) Consider a group replacement model. There are N items in the group. Replacement is made after every t time periods. Assume that all the failures in a group are replaced at the end of the period. Further, C_1 is the cost of replacing a unit in the group, and C_2 is the cost of replacing a failure with $C_2 > C_1$. Find an expression for the least cost associated with group replacement.

(ii) The cost of a machine is ₹ 70,000. The data found from experience is as follows :

Year	**1**	**2**	**3**	**4**	**5**
Resale value (₹)	52,000	35,000	22,000	15,000	9000
Cost of spares (₹)	5000	5750	6000	7800	8000
Cost of labour (₹)	24,000	26,000	28,000	32,000	37,000

When should the machine be replaced ?

(C) Attempt any **three** : **3**

(i) Analytic results are taken into consideration before a simulation study so as to

(a) determine the optimum solution.

(b) identify suitable values of the system of parameters.

(c) identify suitable values of decision variables for the specific choices of system parameters.

(d) All of the above.

(ii) When time value of money is considered

(a) costs need to be discounted.

(b) timing of incurrence of costs is important.

(c) the present value factors serve as weights.

(d) All of the above.

(iii) Give two advantages of simulation.

(iv) What is random number interval ?

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1. (A) (i) Discuss Economic Production Quantity model when shortages are not allowed. **14**

(ii) Find the optimal order quantity for which the price breaks are as follows :

Quantity	**Unit Cost (₹)**
$0 \le Q_1 < 50$	10.00
$50 \le Q_2 < 100$	9.00
$100 \le Q_3$	8.00

The monthly demand for the product is 200 units, the cost of holding is 25 % of the unit cost and ordering cost is ₹ 20 per order.

OR

(B) (i) A manufacturer produces three products x, y, and z in lots. The available floor space is 4000 sq. m. The relevant data for the three products is given below :

Item	X	Y	Z
Annual Demand (units/year)	500	400	600
Cost per Unit (in ₹)	30	20	70
Set-up cost/lot (₹)	800	600	1000
Floor Space required (sq. m.)	5	4	10

The inventory carrying charges for the shop are 20% of the average inventory valuation per annum for each item. If no stock-outs are allowed, determine the optimal lot-size for each item.

(ii) An ice-cream company sells ice-cream by weight. If the product is not sold on the day it is prepared, it can be sold at a loss of ₹ 0.50 per kg. But there is an unlimited market for one day old ice-cream. On the other hand, the company makes a profit of ₹ 3.20 on every kg of the ice-cream sold on the day it is prepared. Past daily orders form a distribution with

$f(x) = 0.02 - 0.0002x,\ 0 \le x \le 100$

How many kgs of ice-cream should company prepare every day ?

(C) Attempt any **four** : **4**

(i) The inventory is __________ stock for an organization.

(a) physical (b) buffer (c) safety (d) chemical

(ii) Total cost of an inventory system does not consider ________ when discounts are not offered.

(a) holding cost (b) shortage cost (c) purchase cost (d) ordering cost

(iii) The classical EOQ model assumes

(a) Lead-time is non-zero (b) Lead-time is zero.

(c) Cycle time is fixed (d) Cycle time is zero.

(iv) The time of placing an order is known as

(a) cycle time (b) lead-time

(c) horizon time (d) None of the above

(v) A company has a demand of a product of 40 items per month. The purchase cost is ₹ 6 per item and the cost of ordering the material is ₹ 15 per order. The holding charge fraction is 20 % per annum. The order should be kept at every

(a) 3 months (b) 4.36 months. (c) 5 months (d) 4 months.

(vi) When cycle time is prescribed, one should not consider ________, in order to calculate total cost of an inventory system.

(a) purchase cost (b) holding cost (c) shortage cost (d) ordering cost

2. (A) Derive (only) steady-state probabilities for $((M / M / c) : (\infty / FCFS))$. **14**

OR

(B) Find expected number of customers in the system for $((M / M / 1) : (\infty / FCFS))$.

(C) Attempt any **four** : **4**

(i) Define bulking.

(ii) When congestion occurs in a queue ?

(iii) What do you mean by "idle server" ?

(iv) Only state different possibilities for the n-customers in the system $((M / M / 1) : (\infty / FCFS))$.

(v) Define waiting time of a customer in the system.

(vi) Give an example of queueing system when customer has to pass through k-phases to quit the system.

3. (A) The data on the operating costs per year and resale price of an equipment A whose purchase price is ₹ 10,000 are given below : **14**

Year	1	2	3	4	5	6	7
Operating cost (₹)	1,500	1,900	2,300	2,900	3,600	4,500	5,500
Resale value (₹)	5,000	2,500	1,250	600	400	400	400

(1) What is the optimum period for replacement ?

(2) When equipment A is 2 years old, equipment B, which is a new model for the same usage is available. The optimum period for replacement is 4 years with an average cost of ₹ 3,600. Should we change equipment A with B ? If so, when ?

OR

(B) We have 4 jobs each of which has to go through six machines in the order M_1, M_2, ..., M_6. Processing time (in hours) is given below :

Job	**Machines**					
	M_1	M_2	M_3	M_4	M_5	M_6
A	18	8	7	2	10	25
B	17	6	9	6	8	19
C	11	5	8	5	7	15
D	20	4	3	4	8	12

Determine a sequence of these four jobs that minimizes the total elapsed time and idle time of each machine.

(C) Attempt any **three** : **3**

(i) What do you mean by group replacement policy ?

(ii) Define discounted factor.

(iii) For 2-jobs on n-machines, state the rules of determining the order.

(iv) Discuss algorithm to find the order of 4 jobs on 5 - machines, when total time taken on machines 2, 3 and 4 is same.

4. (A) Dentist schedules all patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time actually needed to compute the work. **14**

Category	**Time required (in minutes)**	**Probability**
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Check-up	20	0.20

Simulate the dental clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients reach at the clinic at exactly their scheduled arrival time starting at 9:00 am. Use the following random numbers in handling the dentist's clinic :

12 62 83 43 52 24 37 76

OR

(B) The investment company wants to study the investment projects based on the market demand, profit and the investment required which are independent of each other. Following probability distributions are estimated for each of these three factors :

Demand (in 00)	25	30	35	40	45	50	55
Probability	0.05	0.10	0.20	0.30	0.20	0.10	0.05

Profit/unit	3.00	5.00	7.00	9.00	10.00
Probability	0.15	0.20	0.40	0.20	0.10

Investment required (in 00)	2750	3000	3500
Probability	0.05	0.10	0.20

Using simulation process, repeat the trial 10 times. Compute the investment on each trial taking these factors into trials. What is the most likely return ? Use following random numbers :

(30, 12, 16), (59, 09, 69), (63, 94, 26), (27, 08, 74), (64, 60, 61), (28, 28, 72), (31, 23, 57), (54, 85, 20), (64, 68, 18), (32, 31, 87)

In the bracket, the first random number is for annual demand, the second is for profit and the last one is for the investment required.

(C) Attempt any **Three** : **3**

(i) Simulation is based on

(a) Prime numbers (b) Even numbers

(c) Random numbers (d) Odd numbers

(ii) Simulation gives

(a) Optimal solution (b) Sub-optimal solution

(c) Win-win solution (d) Crude solution

(iii) While simulating the experiment.

(a) Probability is to be considered.

(b) Cumulative probability is to be considered.

(c) Random number intervals are to be considered.

(d) None of the above

(iv) Simulation is an experiment which

(a) Saves time (b) Saves time and cost

(c) Saves cost (d) Random process