Seat No. : _____

MD-121

March-2019

BCA., Sem.-I

CC-104 : Basics of Mathematics (BM)
(Old)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (i) Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A ={1, 2, 3, 4} and B = {2, 4, 6, 8}. Answer the following questions. **7**

(1) A' (' is the symbol for complement)

(2) $A \cup B$

(3) $A \cap B$

(4) $A - B$

(5) $A \Delta B$

(6) (B')'

(7) $A' \cup B'$

(ii) Let A = {1, 2, 3, 4} and B = {2, 4, 6, 8} and a function $f:A \to B$ defined as $f(x) = 2x$ and $g:B \to A$ defined as $g(x) = \frac{x}{2}$. Answer the following questions. **7**

(1) Range of f.

(2) Range of g is equals to co-domain of g. (True/False)

(3) Is fog = gof ?

(4) Is f a one-one function ?

(5) What will be the domain of the function fog ?

(6) Find (f+g)(2).

(7) Find $\frac{f(1) + g(2)}{3}$

OR

(i) Let $U = \{x \in Z \mid x^2 \leq 10\}$, $A = \{x \in U \mid x \leq 0\}$ and $B = \{x \in U \mid x \geq 0\}$. Answer the following questions.

(1) $A \cup B$

(2) $(A \cup B)'$

(3) $A' \cap B'$

(4) $U - A$

(5) $A - U$

(6) U'

(7) $A \cap B$

(ii) If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$, then verify that $x^2 + y^2 = 11xy$, $(x > y > 0)$.

(B) Do as Directed : (Any **Four**) **4**

(i) If A is a singleton set, then its power set is empty. (True / False)

(ii) Every set has proper subset. (True / False)

(iii) Every function is one-one and onto. (True / False)

(iv) Every function has finite range. (True / False)

(v) Let a function $f:Z \to Z$, $f(x) = |x|$. The range R_f is equals to

(a) $R_f = N$ (b) $R_f = Z$

(c) $R_f = N \cup \{0\}$ (d) None of these

(vi) A function is onto if,

(a) $R_f = \varnothing$ (b) R_f = Domain of f

(c) f is one-one (d) None of these

2. (A) (i) Let $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix}$. Answer the following questions. **7**

(1) $A + B$

(2) $A - 2B$

(3) $A^T + B^T$

(4) AB

(5) $(A + B)^T$

(6) Rank of A

(7) Verify $(A + B)^T = A^T + B^T$

(ii) Solve the following system using inversion Method : **7**

$x + y + z = 3$

$x + 2y + 3z = 6$

$3x + y + 2y = 6$

OR

(i) Let A be a square matrix given below. Prove that A can be written as a sum of symmetric and skew-symmetric matrices.

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 5 & 6 \\ 1 & 1 & 2 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 2 & -1 \\ 3 & -2 & 1 \end{bmatrix}$

Then verify that $(A + B)^2 = A^2 + 2AB + B^2$

(B) Do as Directed : (Any **Four**) **4**

(i) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices. Then A = B if

(a) $a_{ij} = 0$, for i=j (b) $a_{ij} = b_{ij}$ for all i, j

(c) $A^T = B$ (d) None of these

(ii) Rank of a Null Matrix of order 3 is

(a) 1 (b) 2

(c) 3 (d) 0

(iii) If the determinant of a matrix A is 0, then the determinant of A^T is equals to

(a) 2 (b) –2

(c) 0 (d) None of these

(iv) Every symmetric matrix is a square matrix. (True / False)

(v) Every diagonal matrix is a symmetric matrix. (True /False)

(vi) Inverse of an identity matrix is itself. (True / False)

3. (A) (i) Let two lines $l_1 : 11x - y + 1 = 0$ and $l_2 : 6x + 5y - 4 = 0$. Answer the following questions. **7**

(1) Find m_1 = Slope of a line l_1.

(2) Find m_2 = Slope of a line l_2.

(3) Are l_1 and l_2 parallel lines ?

(4) What will be the slope of a line perpendicular to a line l_2 ?

(5) Find the angle between two lines l_1 and l_2.

(6) Find the intersection point of two lines l_1 and l_2.

(7) Give the line passing through intersection point of above two lines and (0, 0).

(ii) Show that the points (6, 6), (2, 3) and (4, 7) are the vertices of a right angled triangle. Also find the area of this triangle. **7**

OR

(i) Answer the following questions :

(1) Find the distance between two points (1, 2) and (5, 2).

(2) What is the slop of a line parallel to x-axis and passing through (3.14, 2,75) ?

(3) What is the general form of the equation of a line passing through (0, 0) ?

(4) Give the equation of a line passing through the two points (1, 1) and (3, 3).

(5) What is the relation between the slopes of two lines if they are perpendicular ?

(6) What is the equation of a line with slope 2 and y intercept 2.

(7) Are three points (2, 3), (5, 8) and (7, 4) collinear ?

(ii) Find the equation of two lines passing through the point (2, -1) making an angle of 45° with the line $6x + 5y - 1 = 0$.

(B) Do as Directed : (Any **Three**) **3**

(i) Two line y = 3 and y = 6 are,

(a) Parallel (b) Perpendicular

(c) Making angle of 45° (d) None of these

(ii) Equation of a line passing through (0, 0) and having slop m = –1 is.

(a) $x - y = 0$ (b) $x + y = 0$

(c) $x - y = 1$ (d) $x + y = 1$

(iii) Angle between two lines x = 1 and y =1 is 45°. (True / False)

(iv) An equation of a line passing through (0, 0) and having slop 1 is $x - y + 1 = 0$. (True / False)

(v) Two lines $2x + 5y - 3 = 0$ and $4x + 10y - 6 = 0$ are perpendicular. (True / False)

4. (A) (i) Let a function $y = f(x) = 3x^2$. Find the limit of $f(x)$ if $x \to 0$ and also find $\frac{d^2y}{dx^2}$. **7**

(ii) Let $y = e^{2x+3}$. Find $\frac{dy}{dx}$ and $\int y\, dx$. **7**

OR

(i) Find : $\frac{d^2}{dx^2}\left(\int\left(x + \frac{1}{x}\right)dx\right)$

(ii) Evaluate : $\int_{-1}^{1}\left(3x^2 + \frac{1}{4x+5} + 2^x\right)dx$

(B) Do as Directed : (Any **Three**) **3**

(i) The derivative of y = 5 with respect to x is zero. (True / False)

(ii) The derivative of y = 1+ 2 + ... + 10, with respect to x is zero. (True / False)

(iii) Integration of a constant function $f(x) = 1$ is with respect to x is zero. (True / False)

(iv) The area under the curve y = 2 with lower limit $x = 1$ and upper limit $x = 2$ is 2. (True / False)

(v) $\int_{0}^{\log_e 2} e^x\, dx = 1$. (True/False)

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MD-121

March-2019

BCA., Sem.-I

CC-104 : Fundamental Mathematical Concepts (FMC)
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (i) Let $A = \{x \mid x^2 - 7x + 10 = 0\}$, $B = \{x \mid x^2 - 8x + 15 = 0\}$ and $C = \{x \mid x^2 - 9x + 20 = 0\}$. Then verify both the distributive lows. **7**

(ii) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, , then prove that $f\left(\frac{2x}{1-x^2}\right) = 2.f(x)$. **7**

OR

(i) If A = {5, 7}, B = {7, 8}, C = {5, 8}, then verify
$A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(ii) If $p^x = q^y = pq$, then verify that $\frac{1}{x} + \frac{1}{y} = 1$, where $p, q \in R^+ - \{1\}$.

(B) Do as Directed : (Any **Four**) **4**

(i) Let A be singleton set. How many elements in a set P(A) ?

(a) 0 (b) 1

(c) 2 (d) None of these

(ii) The elements of a set $A = \{x \mid x^2 - 1 = 0, x \in Z\}$ are

(a) {1, 2, 3} (b) {–1, 1}

(c) {1, 2} (d) None of these

(iii) If $f:N \to N$, $f(x) = x$, then the domain is

(a) N (b) Z

(c) $N \cup \{0\}$ (d) None of these

(iv) If $f: N \to Z$, $f(x) = x - 2$, then R_f = the range of the function f is.

(a) N (b) $N \cup \{0\}$

(c) $N - \{1\}$ (d) $Z - \{0\}$

(v) A function If $f: N \to N$, $f(x) = x$ is one-one function. (True / False)

(vi) A relation If $f: N \to N$, $f(x) = x - 2$ is a function. (True / False).

2. (A) (i) Solve the following system using Matrix inversion method. **7**

$$2x - 2y + z = 1$$
$$x + 2y + 2z = 2$$
$$2x + y - 2z = 7$$

(ii) Define rank of a matrix. Also find the rank of following matrices : **7**

$$A = \begin{bmatrix} -1 & 2 & 2 \\ -3 & 2 & -2 \\ 1 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 6 \end{bmatrix}$$

OR

(i) For the following two matrices A and B verify that $(A + B)^T = A^T + B^T$.

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$$

(ii) If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, verify that $A^2 - 7A - 2I = 0$, where I is of order 2.

(B) Do as Directed : (Any **Four**) **4**

(i) Let $A = [a_{ij}]$ be a Square Matrix. Then it a scalar matrix if

(a) $a_{ij} = k$ for all i=j, $k \in R$ (b) $a_{ij} = 1$ for all i and j

(c) $a_{ij} = k$ for all $i \neq j$, $k \in R$ (d) None of these

(ii) Rank of an identity matrix of order n is

(a) 1 (b) n

(c) 0 (d) None of these

(iii) Any matrix A can be written as a symmetric and skew symmetric matrices. (True / False)

(iv) For any two matrices A and B, $A^TB^T = B^TA^T$. (True / False)

(v) All diagonal entries of a Skew-symmetric matrix is zero. (True /False)

(vi) Inverse of an identity matrix is itself. (True/False)

3. (A) (i) Determine x so that 5 is the slope of the line through $(x, 12)$ and $(3, 2)$. **7**

(ii) Prove that the points (4, 3), (7, –1) and (9, 3) are the vertices of an isosceles triangle. **7**

OR

(i) If a point P(1, 2) divides a line segment joining points A (–2, –1) and B in the ratio 2 : 3, then finds the co-ordinates of point B.

(ii) Find the equation of a line passing through the intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$ and

(1) having Slope – 1/2.

(2) Is perpendicular to $3x - 2y + 11 = 0$.

(3) Is parallel to $2x - 5y + 13 = 0$.

(4) having x-intercept 2.

(B) Do as Directed : (Any **Three**) **3**

(i) Two line $x = 3$ and $y = 3$ are,

(a) Parallel (b) Perpendicular

(c) Making angle of 45° (d) None of these

(ii) An equation of a line passing through (0, 0) and making angle 0° with x-axis is

(a) $x - y = 0$ (b) $x + y = 0$

(c) $x - y = 1$ (d) None of these

(iii) There points (1, 1), (2, 2) and (3.14, 3.14) are collinear. (True / False)

(iv) Equation $x + y = 0$ passes through (0, 0). (True / False)

(v) Two lines are parallel if $m_1 = m_2$. (True / False)

4. (A) (i) Evaluate following limit : $\lim_{x \to 1} \frac{x^5 - 1}{x^3 - 1}$ **7**

(ii) Find $\frac{dy}{dx}$ when $y = \log(3x - 5)^3$. **7**

OR

(i) Find $\frac{dy}{dx}$ for given $y = e^{2x+3} + (2x + 3)^5$.

(ii) Evaluate : $\int (3x + 4)^5 \, dx$.

(B) Do as Directed : (Any **Three**) **3**

(i) The derivative of $y = (x + x + x)$ is

(a) 0 (b) 1

(c) 2 (d) None of these

(ii) As $x \to 0$, the constant function $f(x) = 3$, tends to,

(a) 3 (b) 0

(c) ∞ (d) None of these

(iii) Integration of a function $f(x) = 2$ is,

(a) 0 (b) 1

(c) 2 (d) None of these

(iv) $\frac{d}{dx}\left(\int_0^1 (x^2 + 3) \, dx\right) =$ _____.

(a) 0 (b) 1

(c) 2 (d) None of these

(v) The derivative of the following function y with respect to x is,

$$y = 1 + \cfrac{2}{3 + \cfrac{4}{5 + \cfrac{6}{7 + \frac{8}{9}}}}$$

(a) 0 (b) 1

(c) 2 (d) None of these