Seat No. : _______________

JG-122

January-2021

B.Sc., Sem.-V

CC-301 : Mathematics

(Linear Algebra – II)

Time : 2 Hours] **[Max. Marks : 50**

Instruction : (1) Attempt any **three** questions from questions **1** to **8**.
(2) Question **9** is compulsory question.
(3) Notations are usual everywhere.
(4) The Figures to the right indicate marks of the question/sub-question.

1. (A) Define a linear functional and show that the trace function tra : $\mu_{n,n} \to R$ defined as tra A = $\sum_{i=1}^{n} a_{i,i}$, for each square matrix A = $(a_{ij}) \in \mu_{n,n}$ is a linear functional. **7**

   (B) If a linear map T : $V_3 \to V_3$ is defined as $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 + e_3$, $T(e_3) = e_3 + e_1$, then solve the operator equation $T(x_1, x_2, x_3) = (6, 8, 4)$. **7**

2. (A) State and prove and dual basis existence theorem. **7**

   (B) Find the dual basis of the basis B = {(1, 0, 0), (1, 1, 0), (1, 1, 1)} for the vector space V_3. **7**

3. (A) Prove that any orthogonal set of non-zero vectors is linearly independent. **7**

   (B) If for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ the map <,> is defined as

   $<x, y> = x_1[y_1 - y_2] + x_2[2y_2 - y_1]$ then show that <,> is an inner product on R^2. **7**

4. (A) Define an orthogonal linear map. **7**
   If (V, <, >) is an inner product space then prove that a linear map T : V → V is an orthogonal linear map **if and only if** $\|T(x)\| = \|x\|$ for all $x \in V$.

   (B) Apply the Gram-Schmidt orthogonalization process to the basis

   B = {(0, –1, 1), (1, 0, –1), (1, 1, 0)} in order to get orthonormal basis for V_3. **7**

5. (A) If det : $V^n \to R$ is a function satisfying the properties of the determinant then prove the followings : **7**

(i) $\det(v_1, v_2,\ldots v_i,\ldots,v_j,\ldots v_n) = \det(v_1, v_2,\ldots,v_i + 3v_j,\ldots,v_j,\ldots v_n)$

(ii) $\det(v_1, v_2, v_3, v_4,\ldots v_n) = -\det(v_1, v_3, v_2, v_4,\ldots, v_n)$.

(B) If $A = \begin{pmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{pmatrix}$ then compute detA without expansion. **7**

6. (A) State and prove the Cramer's rule for solving a system of linear equations. **7**

(B) Apply the laplace Expansion about the last row to find detA for the matrix. **7**

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 4 & 5 \\ 2 & 3 & 1 & 4 \\ 1 & 0 & -1 & 3 \end{pmatrix}$$

7. (A) Express the characteristic equation of 2×2 matrix A in terms of Trace of A and det A. Also prove that a 2×2 real and symmetric matrix has only real eigen values. **7**

(B) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Also find the modal matrix which diagonalizes A. **7**

8. (A) If $T:V \to V$ is a symmetric linear map and if v_i are eigen vectors of T corresponding to eigen values λ_i, i = 1, 2 with $\lambda_1 \neq \lambda_2$ then prove that v_1 and v_2 are orthogonal vectors. **7**

(B) Identify the quadric in R^3 given by $f(x, y, z) \equiv 4xz + 4y^2 + 8y + 8 = 0$. **7**

9. Answer any **four** of the followings in short : **8**

(A) Define the terms :

(i) The space L(U, V)

(ii) A Bilinear form

(B) Define the terms :

(i) Homogeneous operator equation

(ii) Annihilator of a non-empty subset A of a vector space V.

(C) State the Cauchy-Schwarz inequality and the triangle inequality.

(D) Define the terms :

(i) An orthogonal set

(ii) An orthonormal set

(E) Find the matrix A if $A^{-1} = \begin{pmatrix} 1 & 2 \\ -2 & 4 \end{pmatrix}$.

(F) Find detA without expansion of the determinant if $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 5 & 4 & 3 \end{bmatrix}$.
