Seat No. : _______________

AK-121

April-2022

B.Sc., Sem.-VI

CC-310 : Mathematics

(Graph Theory)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (i) Attempt any **THREE** from Section – **I**.

(ii) Figure on right hand side indicates marks.

(iii) Section – **II** is **compulsory**.

SECTION – I

1. (a) Define the following terms with proper graphs. **7**
   (i) Null Graph
   (ii) Adjacent Edges
   (iii) Degree of vertex
   (iv) Pendent

   (b) Define self-complementary graph with graph. Prove that if G is a self complementary graph with n vertices n is either 4t or 4t+l for some integer t. **7**

2. (a) Let G be a non empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycles. **7**

   (b) Define isomorphism of a graph and give one-one example of isomorphic graph and non-isomorphic graph. **7**

3. (a) If T is a tree with n vertices, then prove that it has precisely n-1 edges. **7**

   (b) Define Tree. Draw graph of six trees with six vertices. **7**

4. (a) Let G be a Graph with n vertices $v_1, v_2, v_3, \dots \dots v_n$ and A denotes the adjacency matrix of G. Let k be any positive integer and A^k denote the matrix multiplication of k copies of A. Then prove that $(i, j)^{th}$ entry of A^k is the number of different v_i-v_j walks in G of length k. **7**

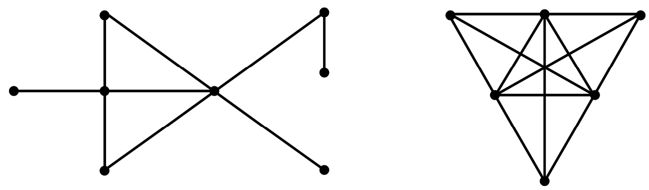
   (b) Draw the graphs having the following matrices as their adjacency matrices. **7**

(i) $$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

5. (a) Define spanning tree. Prove that a graph G is connected if only if it has a spanning tree. **7**

(b) Find κ(G) for the following graph. **7**



6. (a) State and prove WHITNEY'S theorem. **7**

(b) Let e be an edge of the connected graph G then Prove that **7**

(i) e is a bridge if and only if it is in every spanning tree of G.

(ii) e is a loop if and only if it is in no spanning tree of G

7. (a) Prove that a connected graph is Euler if and if G has a cycles $C_{(1)}$, $C_{(2)}$, $C_{(3)}$, ... $C_{(n)}$ such that every edge of G belongs to every cycle $C_{(i)}$. i.e. G is union of edge disjoint cycles. **7**

(b) Discuss Seven bridges problem in graph theory. **7**

8. (a) Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian. **7**

(b) Prove that graph G is Euler if and only if the degree of every vertex is even. **7**

SECTION – II

9. Answer in short : (any **TWO**) **8**

(i) Draw 3-regular graph with 6 vertices.

(ii) Draw Petersen graph.

(iii) How many different Hamiltonian cycles in complete graph K_6 ?

(iv) Discuss whether complete graph K_4 is Euler or not ?

(v) Let G be an acyclic graph 10 vertices 4 connected components how many edges of graph G has ?

(vi) How many different spanning tree of complete graph K_5 ?
