1905E715 Candidate's Seat No : ______

M.Sc. (Sem.-III) Examination
503 Statistics
May-2017

Time : 3 Hours] **[Max. Marks : 70**

STA503 (Multivariate Analysis)

Instructions: 1. All questions carry equal marks.
2. Scientific calculator can be used.

Q-1(a) Let $\underline{x} \sim N_p(\underline{\mu}, \Sigma)$ and let $\underline{x}$, $\underline{\mu}$ and Σ be partition as follows.

$$\underline{x} = \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix}\begin{matrix} r \\ s \end{matrix}, \quad \underline{\mu} = \begin{bmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{bmatrix}\begin{matrix} r \\ s \end{matrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}\begin{matrix} r \\ s \end{matrix}, \; r+s=p.$$

(i) Show that $\underline{x}_1 - \Sigma_{11}\Sigma_{22}^{-1}\underline{x}_2$ and $\underline{x}_2$ are independently distributed.

(ii) Obtain the conditional distribution of $(x_1 / X_2 = x_2)$.

OR

(a) Let $\underline{x}_r$, r =1,2,...k, be independently distributed as $N_p(\underline{\mu}_r, \Sigma_r)$. Then for fixed matrices $A_r : m \times p$, obtain the distribution of $\sum_{r=1}^{k} A_r \underline{x}_r$. If $\underline{\mu}_r = \underline{\mu}$ and $\Sigma_r = \Sigma$; r = 1,2,...,k, then obtain the distribution of $\bar{\underline{x}}$.

(b) Define partial Correlation coefficient. In usual notation obtain the expression in terms of elements of $\Sigma^{-1} = (\sigma^{ij})$ for partial correlation coefficient. Hence obtain $r_{12.3}$.

OR

(b)Define canonical correlation coefficients and canonical variates. In usual notation show that the canonical correlation are solution of the determinant equation

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & -\lambda\Sigma_{22} \end{vmatrix} = 0 .$$

Show that multiple correlation is a special cases of canonical correlation.

Q-2 (a) let $\underline{x}_1, \underline{x}_2, \ldots\ldots, \underline{x}_n$. be n independent observation from $N_p(\underline{\mu}, \Sigma)$ $(n>p)$ Population. Show that the sample mean $\bar{\underline{x}}$ and $S = \sum_{i=1}^{n} (\underline{x}_i - \bar{\underline{x}}).(\underline{x}_i - \bar{\underline{x}})'$ are Independently distributed.

OR

(a) Define Wishart matrix. Obtain probability density function of Wishart matrix V:pxp when $n \geq p$, $\underline{\mu} = 0$ and $\Sigma = I_p$.

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(b) Define Hotelling's T^2 statistic. Show that it is used to test the $H_0 : \underline{\mu} = \underline{\mu}_0$ against $H_1 : \underline{\mu} \neq \underline{\mu}_0$ when $\underline{x} \sim N_p(\mu, \Sigma)$. Obtain the distribution of T^2 under H_0. What is the power of the test?

OR

(b) Show that Hotelling's T^2 can be used to test $H_0 = \rho_{1.23......p} = 0$ against $H_1 = \rho_{1.23......p.} \neq 0$, where $\rho_{1.23......p}$ is multiple correlation coefficient.

Q-3(a) Obtain the estimated minimum ECM rule for classifying an object $\underline{x}_0$ when $\Sigma_1 = \Sigma_2$. Obtain probabilities of errors of misclassification for the classification rule you have obtained.

OR

(a) Define sample Mahalanobis distance D^2, obtain the relation between D^2 and Hotelling's T^2. Hence, obtain the distribution of D^2.

(b) Explain orthogonal factor model with K common factors. Give principal component solution of the factor model.

OR

(b) Obtain null distribution of sample correlation coefficient matrix $R = (r_{ij})$.

Q-4 (a) Explain the technique of One Way MANOVA for the comparison of several multivariate population means.

OR

(a)Define principal components. Write its important applications. If $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ where $\rho > 0$, then find the principal components associated with matrix Σ and find the percentage of total variance explained by first principal component.

(b) Obtain null distribution of the sample correlation coefficient r. Write E(r) and Var(r).

OR

(b)Obtain MLE of $\underline{\beta}$ and σ^2 in GLM. How do you test H: $\underline{c}'\underline{\beta} = \underline{c}'\underline{\beta}_0$ for a specified real vector $\underline{c}'$?

Q-5 Choose the appropriate answer.

1. If $X = \begin{bmatrix} 2 & 8 & 6 & 8 \\ 12 & 9 & 9 & 10 \end{bmatrix}$ is an observation matrix of order 2x4 then its mean vector is
   (A) (6,10) (B)(24,40)
   (C) (14,17,15,18) (D) (14,17,15,18)/2
2. If x:2x1 is distributed as $N_2(\underline{\mu}, \Sigma)$ with $\mu' = (1,5)$ and $\Sigma = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, then the distribution of c'x, where C'=(1,-1) is

(A) $N(-4,2)$ (B) $N_2(\underline{\mu},\Sigma)$

(C) $N(4,-2)$ (D) $N(-4,3)$

3. Let x_1, x_2 and x_3 be distributed as $N_2(\underline{\mu},\Sigma)$ with $\mu' = (0,0)$ and $\Sigma = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$. Then $E(x_1x_1' + x_2x_2' + x_3x_3')$ is

   (A) 1 (B) 6

   (C) $\Sigma = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ (D) $\Sigma = \begin{bmatrix} 3 & 6 \\ 6 & 15 \end{bmatrix}$

4. In usual notations the formula for partial correlation coefficient $r_{12.3}$ is

   (A) $\frac{\sigma^{12}}{\sqrt{\sigma^{11}\sigma^{22}}}$ (B) $\frac{-\sigma^{12}}{\sqrt{\sigma^{11}\sigma^{22}}}$

   (C) $\frac{-\sigma^{12}}{\sigma^{11}\sigma^{22}}$ (D) $\frac{\sigma^{12}}{\sigma^{11}\sigma^{22}}$

5. Let $\underline{x} \sim N_p(\underline{\mu},\Sigma)$ and consider the partition of $\underline{x}$, $\underline{\mu}$ and Σ as follows.

   $\underline{x} = \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix}$, $\underline{\mu} = \begin{pmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$; where $\Sigma_{11} : rxr$ and $\Sigma_{22} : sxs$ are matrices with r+s=p. The conditional distribution of $\underline{x}_1$ given $\underline{X}_2 = \underline{x}_2$ is

   (A) $N_p(\underline{\mu}_1 + \Sigma_{12}\Sigma_{22}^{-1}(\underline{x}_2 - \underline{\mu}_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}')$

   (B) $N_r(\underline{\mu}_1 - \Sigma_{12}\Sigma_{22}^{-1}(\underline{x}_2 - \underline{\mu}_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}')$

   (C) $N_p(\underline{\mu}_2 + \Sigma_{12}'\Sigma_{11}^{-1}(\underline{x}_1 - \underline{\mu}_1),\ \Sigma_{22} - \Sigma_{12}'\Sigma_{11}^{-1}\Sigma_{12})$

   (D) $N_s(\underline{\mu}_2 - \Sigma_{12}'\Sigma_{11}^{-1}(\underline{x}_1 - \underline{\mu}_1),\ \Sigma_{22} - \Sigma_{12}'\Sigma_{11}^{-1}\Sigma_{12})$

6. If $\underline{x}_1$ and $\underline{x}_2$ are independent $N_p(\underline{\theta}_i,\Sigma_i)$; i=1,2 respectively, then the distribution of $(\underline{x}_1 - \underline{x}_2)$ is

   $(A) N_p(\underline{\theta}_1 - \underline{\theta}_2, \Sigma_1 - \Sigma_2)$ $(B) N_p(\underline{\theta}_1 - \underline{\theta}_2, \Sigma_1 + \Sigma_2)$

   $(C) N_p(\underline{\theta}_1 + \underline{\theta}_2, \Sigma_1 + \Sigma_2)$ $(D) N_p(\underline{\theta}_1 + \underline{\theta}_2, \Sigma_1 - \Sigma_2)$

7. Let $\underline{x} \sim N_p(\underline{\mu},\Sigma)$, where Σ is a nonsingular matrix. The characteristic function of the vector $\underline{y} = C\underline{x}$ is given by

   (A) $\phi_{\underline{y}}(\underline{t}) = \exp\left(i\underline{t}'C\underline{\theta} - \frac{1}{2}\underline{t}'C\Sigma C'\underline{t}\right)$ (B) $\phi_{\underline{y}}(\underline{t}) = \exp\left(i\underline{t}'C\underline{\mu} - \frac{1}{2}\underline{t}'C\Sigma C'\underline{t}\right)$

   (C) $\phi_{\underline{y}}(\underline{t}) = \exp\left(i\underline{t}'C\underline{\mu} + \frac{1}{2}\underline{t}'C\Sigma C'\underline{t}\right)$ (D) $\phi_{\underline{y}}(\underline{t}) = \exp\left(i\underline{t}'C\underline{\theta} + \frac{1}{2}\underline{t}'C\Sigma C'\underline{t}\right)$

8. The unbiased estimate of the variance covariance matrix for multivariate normal distribution is

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$(A)\sum_{i=1}^{n}(\underline{x}_i-\bar{\underline{x}})(\underline{x}_i-\bar{\underline{x}})'$ (B) $\sum_{i=1}^{n}(\underline{x}_i-\bar{\underline{x}})(\underline{x}_i-\bar{\underline{x}})'/n$

$(C)\sum_{i=1}^{n}(\underline{x}_i-\bar{\underline{x}})(\underline{x}_i-\bar{\underline{x}})'/(n-1)$ (D) $\sum_{i=1}^{n}(\underline{x}_i-\bar{\underline{x}})'(\underline{x}_i-\bar{\underline{x}})/n$

9. If the joint pdf of (x, y) is $\frac{1}{2.4\pi}\exp\left[-\left\{\frac{(x^2/4)-(1.6xy/2)+y^2}{0.72}\right\}\right]$ then the values of $\mu_x,\mu_y,\sigma_x,\sigma_y$ *and* ρ_{xy} are respectively

(A) (0, 0, 2, 1, 0.8) (B) (0, 0, 2, 1, 0.6)
(C)(1/2, 1, 2, 1, 0.4) (D) (0, 0, 1, 2, 0.8)

10. The Hotelling's T^2 is a generalization of
(A) Chi-square distribution (B) t-distribution
(C) Square of t-distribution (D) F-distribution

11. Let $\underline{X}_1,\underline{X}_2,\ldots,\underline{X}_{20}$ be a random sample of size n=20 from a $N_6(\underline{\mu},\Sigma)$ population. If $B=\begin{bmatrix}1,0,0,0,0,0\\0,0,1,0,0,0\end{bmatrix}$, then the distribution of B* = B(19S)B' is
(A) Chi-square distribution with degrees of freedom 6.
(B) Non-central Chi-square distribution with degrees of freedom 6
(C) $W_2(B^*,19,B\Sigma B')$ (D) $W_6(B^*,19,B\Sigma B')$

12. Let $\underline{X}_1,\underline{X}_2,\ldots,\underline{X}_{25}$ be a random sample of size n = 50 from a $N_6(\underline{\mu},\Sigma)$ population. Then the distribution of $(\underline{x}_1-\underline{\mu})'\Sigma^{-1}(\underline{x}_1-\underline{\mu})$ is

(A) Chi-square with '50' degrees of freedom

(B) Non central Chi-square with '50' degrees of freedom

(C) Chi-square with '6' degrees of freedom

(D) Non central Chi-square with '6' degrees of freedom

13. If the sample mean vector $\bar{\underline{x}}$ and variance covariance matrix S of three iid observations from a bivariate normal distribution are $\begin{bmatrix}8\\6\end{bmatrix}$ and $\begin{bmatrix}4 & -3\\-3 & 9\end{bmatrix}$ respectively then the value of observed T^2 for testing $H_0:\mu=\mu_0=\begin{pmatrix}9\\5\end{pmatrix}$ is

(A) 7/9 (B) 9/7 (C) 7/27 (D) 9/27

14. let $\underline{x}\sim N_p(\underline{\mu},\Sigma)$, $\Sigma>0$. Then the distribution of $(\underline{x}-\underline{\mu})(\underline{x}-\underline{\mu})'$ is
$(A)\chi_p^2(\underline{\mu}'\Sigma^{-1}\underline{\mu})$ (B) $W_p(1,\Sigma)$
(C) $W_p(n,\Sigma)$ (D) χ_p^2