0305E328 Candidate's Seat No : ________

M.Sc. (Sem.-II) Examination

407

Statistics

Time : 3 Hours] **May-2017** **[Max. Marks : 70**

Instructions: 1. All questions carry equal marks.
2. Scientific calculator can be used.

1. (a) Define hazard rate. Discuss different types of hazard rates.

The cdf of the life time(months) of an electronic device is $F(t) = \begin{cases} t^3/216, & \text{if } 0 \le t \le 6 \\ 1, & \text{if } t \ge 6 \end{cases}$

(i) What is the failure rate function of this equipment? (ii) What is the mean time to failure?

(iii) What is the reliability of the device at age 3 months?

OR

(a) Define mean time between failure(mtbf) and mean time to failure(mttf). Obtain relation between reliability and mean time between failure. If the reliability function of a system is R(t) = exp(-2 -3t), t > 0, find the failure Rate function and mtbf.

(b) State and prove memoryless property for exponential distribution. Show also that this is the only continuous distribution which possess such a property.

OR

(b) Let $X_{(1)}, X_{(2)}, \ldots\ldots, X_{(n)}$ are ordered failure times of n components having iid exponential life time distribution with mean $\theta > 0$. Obtain the distribution of k^{th} gap $G_k = X_{(k)} - X_{(k-1)}$, k = 1 ,2, … ,n with $X_{(0)} = 0$. Also find mean and variance of $X_{(k)}$

2. (a) What is censoring? Why censoring is used I life test. Let life time model be exponential with hazard rate $1/\theta$. Obtain MLE of the hazard rate under type-I censoring without replacement.

OR

(a) Discuss type-II censoring without replacement. Under this scheme obtain MLE of location parameter μ and scale parameter θ, $\theta>0$, in case of two – parameter exponential distribution and their variances.

(b) Discuss type-I censoring. Under this scheme obtain MLE of parameter θ^2 for Rayleigh life time Model. Is it unbiased estimator for θ^2? If, not, hence obtain unbiased estimator. Also obtain variance of the MLE for θ^2.

OR

(b) Discuss Type – II censoring with replacement for exponential life time model. Obtain MLE and UMVUE for reliability at time t under this censoring scheme.

(P.T.O)

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3. (a) Obtain reliability of a (i) series (ii) parallel and (iii) series-parallel systems when all the components are independent and have constant failure rate $\theta>0$.

OR

(a) Consider a machine with three components whose time to failure are independently distributed as random variable with mean $\theta > 0$. The machine continuous to work as long as at least one component work. Find the expected time to failure of the machine. Find the probability that the machine will survive at least t_0 time.

(b) Let the failure time distribution is exponential with mean life time θ, $\theta>0$. Only the number of failures(m) among the n devices during the first t_0 hours of operation is recorded, $m < n$. Determine the MLE of θ and its asymptotic standard error.

OR

(b) Let the failure time distribution is exponential with mean life time θ, $\theta>0$. Only the Number of failures(m) among the n devices during the first t_0 hours of operation under with replacement case is recorded, $m < n$. Determine the MLE and UMVUE of R(t).

4. (a) What is Bayes estimation? Discuss normal and extensive forms of Bayesian estimation. Obtain general form of the Bayes estimator of any parameter under squared error loss function.

OR

(a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from normal N(θ, 1) distribution. Suppose that an estimator for θ is given by $T = c\bar{x}$, $\bar{x} = \sum_{i=1}^{n} x_i$. Determine constant c such that it becomes a Bayes estimator under a squared error loss function. Obtain its Bayes risk.

(b) Using a random sample of size n having binomial b(k, p), $0 < p < 1$ derive Bayes estimator of p/(1-p) under weighted squared error loss function with weight $w(p) = p^3$ when prior distribution of p is beta $\beta(a,b)$. Also find posterior variance.

OR

(b) Consider a random sample of size n from Poisson distribution with mean $\lambda > 0$. Suppose that the prior distribution of λ is gamma G(p), $p > 0$. Obtain Bayes estimator of λ under squared error loss function. Also obtain its Bayes risk.

5 . Attempt the following:

i. State the reliability of the exponential life time model with hazard rate $\theta>0$.

ii. State the general form of the likelihood function under Type-II censoring WOR.

iii. State the distribution of number of failures under type-I censoring without replacement for exponential life time model.

iv. State the distribution of number of failures under type-I censoring with replacement for exponential life time model.

v. If two components are connected in parallel system having equal reliability 0.7, find the system reliability.

vi. If $X_{(1)}, X_{(2)}, \ldots\ldots, X_{(n)}$ are ordered life times of n units on the test. If the life time distribution is exponential with mean $1/\theta$, state the distribution of $Z_{(i)} = (n-i+1)(X_{(i)} - X_{(i-1)}), i = 1,2, \ldots, n$.

vii. Define cumulative hazard function.

viii. State the name of the life time model which has strictly increasing hazard rate.

ix. For life test based on 20 items the test was terminated after the 5 failures were observed under without replacement. If the observed total test time is 230 hours, find mle of the mean life time in case of exponential life time model.

x. State the expression to find pdf of a life time variate from its hazard function.

xi. you have two components having reliability 0.05. Suggest appropriate system based on these two components.

xii. Define prior distribution.

xiii. Define weights squared error loss function.

xiv. Define risk function.

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Note: (i) Attempt all questions. (ii) All questions carry equal marks.

Q.1 (a) Describe the revised simplex method for solving a linear programming problem.

OR

(a) Show that if either the primal or the dual problem has a finite optimal solution, then the other one also possess the same, and the optimal values of the objective functions of the two problems are equal, Max. Z_x = Min. Z_y.

(b) Discuss sensitivity analysis with respect to addition of new variable.

OR

(b)Discuss the role of sensitivity analysis in linear programming. Under what circumstances is it needed, and under what conditions do you think it is not necessary?

Q.2 (a) Discuss sensitivity analysis with respect to change in the objective function coefficient c_j.

OR

(a) Discuss parametric linear programming with respect to variation in the objective function coefficients.

(b) Discuss sensitivity analysis with respect to addition of new constraint.

OR

(b) Explain the method of solving a zero-sum two person game as a linear programming problem.

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Q.3 (a) What is integer linear programming? Explain Gomory's all integer cutting plane method.

OR

(a) State the principle of optimality in dynamic programming. Describe the basic features which characterize a dynamic programming problem.

(b) Explain fractional programming with suitable examples.

OR

(b) Discuss zero-one integer programming with examples.

Q.4 (a) Explain graphical solution method for goal programming.

OR

(a) State some problem areas in management where goal programming might be applicable.

(b) Explain alternative simplex method for goal programming.

OR

(b)Explain the terms: (i) Deviational variables (ii) Preemptive priority factors (iii) Differential weights.

Q.5 Answer the following:

(i) Revised simplex method automatically generates the inverse of the current basis matrix and the new basic feasible solution as well.

(a) True (b) False.

(ii) Addition of an additional constraint in the existing constraints will cause a

(a)change in objective function coefficients c_j (b) change in coefficients a_{ij}

(c)both (a) and (b) (d) none of the above.

(iii) If either the primal or the dual LP problem has an unbounded objective function value, then the other problem has no feasible solution.

(a) True (b) False.

(iv) When an additional variable is added in the LP model, the existing optimal solution can further be improved if

(a) $c_j - z_j \geq 0$ (b) $c_j - z_j \leq 0$ (c) both (a) and (b) (d) none of the above.

(v) Game theory models are classified by the

(a) number of players (b) sum of all payoffs (c) number of strategies (d) all of the above.

(vi) What happens when maximin and minimax values of the game are same?

(a) no solution exists (b) solution is mixed (c) saddle point exists (d) none of the above.

(vii)The size of the payoff matrix of a game can be reduced by using the principle of

(a)game inversion (b) rotation reduction (c) dominance (d) game transpose.

(viii) A non-integer variable is chosen in the optimal simplex table of the integer LP problem to

(a) leave the basis (b) enter the basis (c) to construct a Gomory cut (d) none of the above.

(ix) The situation of multiple solutions arises with

(a) cutting plane method (b) branch and bound method
(c) both (a) and (b) (d) none of the above.

(x) While applying the cutting-plane method, dual simplex is used to maintain
(a)optimality (b) feasibility (c) both (a) and (b) (d) none of the above.

(xi) Deviational variables in GP model must satisfy the following conditions:

(a) $d_i^- \times d_i^+ = 0$ (b) $d_i^+ - d_i^- = 0$ (c) $d_i^+ + d_i^- = 0$ (d) none of the above.

(xii) In GP problem, a goal constraint having over achievement variable is expressed as a

(a) $\geq$ constraint (b) $\leq$ constraint (c) = constraint (d) all of the above.

(xiii) If the largest value of each goal in the 'solution-value $\mathbf{X_B}$'column is zero, then it indicates

(a) multiple solution (b) infeasible solution (c) optimal solution (d) none of the above.

(xiv) Dynamic programming approach

(a) optimizes a sequence of interrelated decision over a period of time

(b) provides optimal solution to single period decision-problem

(c) provides optimal solution to long-term corporate planning problems

(d) all of the above.

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