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Candidate's Seat No : _____

M.Sc. (Sem.-II) Examination

408

Statistics

May-2017

Time : 3 Hours]

[Max. Marks : 70

STA 408 (Distribution Theory)

Instructions: 1. All questions carry equal marks.

2. Scientific calculator can be used.

Total Marks: 70

Q-1 (a) Define Contagious Distribution. Write applications of Contagious Distribution.

If X_1, X_2, \dots, X_N are N identically independently distributed random variables and N is also a random variable independent of X_i 's then show that

$$(i) E(S_N) = E(N)E(X)$$

$$(ii) V(S_N) = E(N)V(X) + V(N)\{E(X)\}^2, S_N = \sum_{i=1}^N X_i$$

OR

(a) Define Neyman type A distribution. Obtain its probability generating function.

Estimate the parameters of the Poisson Poisson distribution using method of moments.

Also obtain recurrence relation for this distribution.

(b) Define Poisson – Binomial distribution. Obtain its probability generating function. Show that Poisson – Binomial distribution tends to Poisson – Poisson distribution. State necessary assumptions involved.

OR

(b) Define Poisson – Pascal distribution. Obtain recurrence relations for Probabilities and descending factorial moments for this distribution.

Q-2(a) Discuss the roll of non-central distributions in statistical inference. If $X \sim N(\mu, 1)$ then, obtain probability density function of non-central Chi-square distribution using M.G.F.

OR

(a) Define non-central 'F' distribution with (n_1, n_2) degrees of Freedom. In usual notations obtain probability density function of non-central 'F' distribution.

(b) Define non-central 't' statistic. In usual notations obtain probability density function of non-central 't' distribution.

OR

(b) State and prove the relation between non-central chi-square, non-central F and non-central t distributions.

Q-3 (a) Obtain the joint probability density function of the r th and s th ordered statistics. Hence write the joint pdf of largest and the smallest order Statistics.

OR

(P.T.O)

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(a) Let a random variable 'X' follows an Exponential distribution with mean θ , $\theta > 0$. If a random sample of size n is taken from this distribution then show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independently distributed.

(b) If a random sample of size 'n' is taken from the exponential distribution with mean 1/3 then find the probability that the sample range does not exceed 2.

OR

(b) Obtain the distribution of sample median when (i) n is odd number and (ii) n is even number.

Q-4 (a) If $X_{(n)} = \max \{X_1, X_2, \dots, X_n\}$ then show that $E(X_{(n)}) =$

$$E(X_{(n-1)}) + \int_0^{\infty} F^{(n-1)}(x)(1 - F(x)) dx.$$

Find $E(X_{(r)})$ for $F(x) = 1 - e^{-\theta x}$; $\theta > 0, x \geq 0$.

OR

(a) Explain the procedure of obtaining Confidence Interval for p^{th} Quantile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cumulative distribution function $F_X(x)$ then obtain

$$P(X_{(3)} < \text{Population median} < X_{(5)})$$

(b) Define rank-order statistics with appropriate example. Give functional definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

OR

(b) Obtain the correlation coefficient between r^{th} and s^{th} order statistics for the uniform distribution $U(0,1)$. Hence write the correlation coefficient between the smallest and the largest order statistics.

Q-5(a) State whether the following statements are true or false. Justify your answer.

1. When ' $v=1$ ', student's t distribution tends to Cauchy distribution.
2. For Poisson-Pascal distribution mean is less than variance.
3. If $np\lambda = \theta_1$ and $p \rightarrow 0$ (or $\lambda \rightarrow \infty$) then Poisson-Pascal distribution tends to Negative Binomial distribution.

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4. The correlation coefficient between the smallest and the largest order statistics for the uniform distribution $U(0,1)$ is $1/n$.
5. If $X \sim N(\mu, 1)$ and Y is an independent chi-square variate with n degrees of freedom then $t = X / \sqrt{Y/n}$ has a central t distribution.
6. In probability generating function if we put ' $z=(1-t)^{-1}$ ' then we get ascending factorial moment generating function.
7. If X_1, X_2, \dots, X_n are n independent observations then the order statistics Y_1, Y_2, \dots, Y_n are independent order statistics.
8. Rank-order statistics are invariant under monotone transformation.
9. The i th rank-order statistics $r(X_i)$ is the rank of the i th observation in the original ordered sample.

(b) Answer the following questions.

1. Write the distribution of smallest ordered statistic.
 2. Define descending factorial moment generating function.
 3. Write mean and variance of r th ordered statistic for $U(0, 1)$ distribution.
 4. Write the recurrence relation for the probability for the Neyman type-B Distribution.
 5. If a random sample of size '5' is taken from uniform distribution $U(0,1)$ then write the probability density function of the sample median.
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