1304E120 Candidate's Seat No :________

M.Sc. (Sem.-IV) Examination

508

Mathematics

Time : 3 Hours] **April-2017** **[Max. Marks : 70**

Q.1. (A) Attempt any one [7]

(1) State and prove the uniqueness theorem for the real and complex valued continuous functions only.

(2) Prove that the set of all trigonometric polynomials is dense in C and in L^p for $1 \leq p < \infty$.

(B) Attempt any two [4]

(1) Show that the Fourier transform map $T : L^1 \to l_\infty(Z)$ is linear and continuous.

(2) If f is absolutely continuous then show that $\widehat{Df}(n) = in\hat{f}(n)$.

(3) If a trigonometric series converges uniformly then show that it is a Fourier series.

(C) Answer in brief [3]

(1) Define convolution in L^1.

(2) For f in L^∞, define its essential-sup norm.

(3) Verify the Riemann-Lebesgue lemma for the function

$$f(x) = 10e^{i2x} - 100ie^{-i2x} + 1000.$$

Q.2. (A) Attempt any one [7]

(1) Let $\{K_n\}$ be an approximate identity. Then show that

$$\lim_{n\to\infty} \|K_n * f - f\|_\infty = 0, \quad \forall f \in C.$$

(2) Let $f \in L^1$. Show that

(i) if $g \in C^1$, then $f * g \in C^1$;

(ii) if g is absolutely continuous then $f * g$ is absolutely continuous.

(B) Attempt any two [4]

(1) Let $1 \leq p \leq \infty$ and q be the conjugate index of p. If $f \in L^p$ and $g \in L^q$ then show that $f * g$ is continuous.

(2) Prove that

$$T_a(f * g) = T_a f * g = f * T_a g.$$

(3) If $f(x) = 10e^{i2x} - 100ie^{-i2x} + 1000$, determine the function $(f * f)(x)$.

(C) Answer in brief [3]

(1) Define the term "approximate identity ".

(2) Let γ be a complex continuous algebra homomorphism of L^1. If $f(x) = e^{i2x} + 2$, $g(x) = e^{ix} + 1$ and $\gamma(f) = 1$ then what is the value of $\gamma(g)$?

P.T.O.

E120-2

(3) True or False: L^1 has zero divisors with respect to convolution.

Q.3. (A) Attempt any one [7]

(1) If $f \in L^1$, then prove that

$$\int_a^b f(x)dx = \hat{f}(0)(b-a) + \sum_{n\neq 0} \hat{f}(n)\frac{e^{inb}-e^{ina}}{in}.$$

(2) Show that

$$\|D_N\|_1 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |D_N(x)|dx = \frac{4}{\pi^2}(logN) + O(1).$$

(B) Attempt any two [4]

(1) Show that the sequence of Fejer kernel forms an approximate identity for convolution.

(2) State (only) Fejer's theorem.

(3) If for a trigonometric series $\sum c_n e^{inx}$, its cesaro means converge in L^1 norm to f, then show that $\sum c_n e^{inx}$ is a Fourier series of f.

(C) Answer in brief [3]

(1) State any one consequence of localisation principle.

(2) Using $S_N f(x) = \sum_{n=-N}^{N} \hat{f}(n)e^{inx}$, express $\sigma_N f(x)$ as a trigonometric polynomial.

(3) True or False: If $a_n = \int_{-\pi}^{\pi} D_n(x)dx$ and $b_n = \frac{a_n}{n+1}$ then (b_n) is a convergent sequence.

Q.4. (A) Attempt any one [7]

(1) If (a_n) is convex and bounded, then prove that (a_n) is decreasing and $n\Delta a_n \to 0$. Further, show that (a_n) is quasi-convex.

(2) Let $a_n \downarrow 0$ and $f(x) = \sum a_n \sin nx$. Show that $f \in L^1$ if and only $\sum \frac{a_n}{n} < \infty$.

(B) Attempt any two [4]

(1) If $a_n \downarrow 0$, then show that $\sum a_n \cos nx$ converges uniformly in $\delta \leq |x| \leq \pi$.

(2) Show that the range of the Fourier transform map is dense in $C_0(Z)$.

(3) Which of the following series are the Fourier series of a continuous function?

(i) $\sum_{n=2}^{\infty} \frac{\sin nx}{n \log n}$.

(ii) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$.

(iii) $\sum_{n=2}^{\infty} \frac{\sin nx}{\log n}$.

(iv) $\sum_{n=-\infty}^{\infty} \frac{e^{inx}}{n^2+1}$.

E 1203

(C) Answer in brief [3]

(1) Is $a_n = \frac{1}{n+1}$ convex?

(2) Define quasi-convex sequence.

(3) True or False: The sequence $a_n = \frac{1}{n+1}$ is of bounded variation.

Q.5. (A) Attempt any one [7]

(1) State the Uniform Boundedness theorem and using it show that there exists a function which is continuous at 0 but whose Fourier series diverges at 0.

(2) Show that $C \subseteq L^1 * C$.

(B) Attempt any two [4]

(1) State (only) some of the consequences of Jordan's theorem.

(2) If $f \in L^1$ then show that $\sum_{n \neq 0} \frac{\widehat{f}(n) e^{inx}}{n}$ converges uniformly.

(3) Let $f \in L^1$ and s be any complex number. If for some positive δ, $\int_0^\delta \frac{|f_s^*(y)|}{y} dy < \infty$, then show that $S_N f(x) \to s$.

(C) Answer in brief [3]

(1) Characterise $f \in L^2$ which can be factorised as $g * h$ with $g, h \in L^2$.

(2) State only Jordan's theorem.

(3) True or False: $L^1 * L^1 = L^1$.