1304E120

Candidate's Seat No :_____

M.Sc. (Sem.-IV) Examination 508

Mathematics

Time: 3 Hours] April-2017 [Max. Marks: 70

Q.1. (A) Attempt any one

[7]

- (1) State and prove the uniqueness theorem for the real and complex valued continuous functions only.
- (2) Prove that the set of all trigonometric polynomials is dense in C and in L^p for $1 \le p < \infty$.
- (B) Attempt any two

[4]

- (1) Show that the Fourier transform map $T:L^1\to l_\infty(Z)$ is linear and continuous.
- (2) If f is absolutely continuous then show that $\widehat{Df}(n) = in\widehat{f}(n)$.
- (3) If a trigonometric series converges uniformly then show that it is a Fourier series.
- (C) Answer in brief

[3]

- (1) Define convolution in L^1 .
- (2) For f in L^{∞} , define its essential-sup norm.
- (3) Verify the Riemann-Lebesgue lemma for the function

$$f(x) = 10e^{i2x} - 100ie^{-i2x} + 1000.$$

Q.2. (A) Attempt any one

[7]

(1) Let $\{K_n\}$ be an approximate identity. Then show that

$$\lim_{n\to\infty} \|K_n * f - f\|_{\infty} = 0, \quad \forall f \in C.$$

- (2) Let $f \in L^1$. Show that
 - (i) if $g \in C^1$, then $f * g \in C^1$;
 - (ii) if g is absolutely continuous then f * g is absolutely continuous.
- (B) Attempt any two

[4]

- (1) Let $1 \le p \le \infty$ and q be the conjugate index of p. If $f \in L^p$ and $g \in L^q$ then show that f * g is continuous.
- (2) Prove that

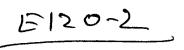
$$T_a(f*g) = T_af*g = f*T_ag.$$

- (3) If $f(x) = 10e^{i2x} 100ie^{-i2x} + 1000$, determine the function (f * f)(x).
- (C) Answer in brief

[3]

- (1) Define the term "approximate identity".
- (2) Let γ be a complex continuous algebra homomorphism of L^1 . If $f(x) = e^{i2x} + 2$, $g(x) = e^{ix} + 1$ and $\gamma(f) = 1$ then what is the value of $\gamma(g)$?

P. T. O.



- (3) True or False: L^1 has zero divisors with respect to convolution.
- Q.3. (A) Attempt any one

[7]

(1) If $f \in L^1$, then prove that

$$\int_a^b f(x)dx = \hat{f}(0)(b-a) + \sum_{n \neq 0} \hat{f}(n) \frac{e^{inb} - e^{ina}}{in}.$$

(2) Show that

$$||D_N||_1 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |D_N(x)| dx = \frac{4}{\pi^2} (log N) + O(1).$$

(B) Attempt any two

[4]

- (1) Show that the sequence of Fejer kernel forms an approximate identity for convolution.
- (2) State (only) Fejer's theorem.
- (3) If for a trigonometric series $\sum c_n e^{inx}$, its cesaro means converge in L^1 norm to f, then show that $\sum c_n e^{inx}$ is a Fourier series of f.
- (C) Answer in brief

[3]

- (1) State any one consequence of localisation principle.
- (2) Using $S_N f(x) = \sum_{n=-N}^{N} \hat{f}(n)e^{inx}$, express $\sigma_N f(x)$ as a trigonometric polynomial.
- (3) True or False: If $a_n = \int_{-\pi}^{\pi} D_n(x) dx$ and $b_n = \frac{a_n}{n+1}$ then (b_n) is a convergent sequence.
- Q.4. (A) Attempt any one

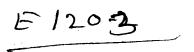
[7]

- (1) If (a_n) is convex and bounded, then prove that (a_n) is decreasing and $n\Delta a_n \to 0$. Further, show that (a_n) is quasi-convex.
- (2) Let $a_n \downarrow 0$ and $f(x) = \sum a_n \sin nx$. Show that $f \in L^1$ if and only
- (B) Attempt any two

[4]

- (1) If $a_n \downarrow 0$, then show that $\sum a_n \cos nx$ converges uniformly in $\delta \leq |x| \leq \pi$.
- (2) Show that the range of the Fourier transform map is dense in $C_0(Z)$.
- (3) Which of the following series are the Fourier series of a continuous function?
 - (i) $\sum_{n=2}^{\infty} \frac{\sin nx}{n \log n}$ (ii) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$

 - (iii) $\sum_{n=2}^{\infty} \frac{\sin nx}{\log n}.$ (iv) $\sum_{n=-\infty}^{\infty} \frac{e^{inx}}{n^2 + 1}$



- (C) Answer in brief
- (1) Is $a_n = \frac{1}{n+1}$ convex?
- (2) Define quasi-convex sequence.
- (3) True or False: The sequence $a_n = \frac{1}{n+1}$ is of bounded variation.
- Q.5. (A) Attempt any one

[7]

- (1) State the Uniform Boundedness theorem and using it show that there exists a function which is continuous at 0 but whose Fourier series diverges at 0.
- (2) Show that $C \subseteq L^1 * C$.
- (B) Attempt any two

[4]

- (1) State (only) some of the consequences of Jordan's theorem.
- (2) If $f \in L^1$ then show that $\sum_{n \neq 0} \frac{\widehat{f}(n)e^{inx}}{n}$ converges uniformly.
- (3) Let $f \in L^1$ and s be any complex number. If for some positive δ , $\int_0^\delta \frac{|f_s^*(y)|}{y} dy < \infty$, then show that $S_N f(x) \to s$.
- (C) Answer in brief

[3]

- (1) Characterise $f \in L^2$ which can be factorised as g * h with $g, h \in L^2$.
- (2) State only Jordan's theorem.
- (3) True or False: $L^1 * L^1 = L^1$.