1605E599

Candidate's Seat No : _____

M.Phil. Science Examination
Paper-III : Mathematics
May-2017

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Write a short note on Platonic Solids. (7)

OR

(a) Write a short note on four dimensional analogues of Platonic Solids. (7)

(b) Answer any <u>two</u> of the following briefly: (4)

(i) For any regular spherical Polyhedron {m, n} write down V, E and F in terms of m, n.

(ii) Calculate the Total Defect of the Generalized Pyramid.

(iii) Calculate the Total Defect of the frame of the Black Board.

(c) Answer all of the following very briefly: (3)

(i) Draw {4, 4} on torus.

(ii) Give an example of non-regular, non-spherical Polyhedron.

(iii) Find V, E, F and C for the Hypercube. What is V - E + F - C?

2. (a) Write a short note on Isometries. (7)

OR

(a) Give complete account of the symmetry group of any regular n-gon. (7)

(b) Answer any <u>two</u> of the following briefly: (4)

(i) Define Frieze Groups and Frieze Patterns.

(ii) Describe the Symmetry Group of a line in the plane.

(iii) Write down the Symmetry Group of the unit circle.

(c) Answer all of the following very briefly: (3)

(i) Give two illustrations of Frieze Patterns.

(ii) Describe the Symmetry group of the first letter of English Alphabet after drawing its Picture.

(iii) Describe the Symmetry group of the last letter of English Alphabet after drawing its Picture.

3. (a) What do you mean by a topological characterization of the topological space? Describe two classical results in this direction. Also narrate one application of this result. (7)

OR

(P.T.O)

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(a) Show that any countable metric space is a subspace of $\mathbb{Q}$. (7)

(b) Answer any <u>two</u> of the following briefly: (4)

(i) Define the phrase 'Locally-compact space'. Give one example of a space which is locally- compact but not compact.

(ii) Explain difference between a non-locally-compact space and a nowhere locally-compact space with genuine illustration.

(iii) Write a short note on the space $2^{\aleph_0}$.

(c) Answer all of the following very briefly: (3)

(i) Give an example of a non-trivial(other than the indiscrete space) non-metrizable space.

(ii) The space $J = \{\frac{1}{n}/n \in \mathbb{N}\}$ as a subspace of $\mathbb{R}$ is completely metrizable. Justify.

(iii) Define a nowhere-dense set giving an illustration.

4. (a) Explain the word "Minor". Show that K_5 and $K_{3,3}$ both are minors of the Petersen Graph. Draw K_5 and $K_{3,3}$ on the torus. (7)

<u>OR</u>

(a) Show that the Polytopal Graph (i.e. 1-skeleton) of the Hyper Cube is a Toroidal Graph. (7)

(b) Answer any <u>two</u> of the following briefly: (4)

(i) Explain the recursive process of defining Polytopes α_n.

(ii) Polyhedral graph of the Octahedron is Planar. Explain.

(iii) 1-skeleton of the Dodecahedron is a planar Graph. Justify.

(c) Answer all of the following very briefly: (3)

(i) What is a general Prism? Draw its Schlegel Diagram.

(ii) Write down V, E and F for the truncated icosahedron.

(iii) Give an example of a Graph which can not be drawn on torus.

5. (a) Give complete account of the Theory of Nets. (7)

<u>OR</u>

(a) Give complete account of the Theory of Filters. (7)

(b) Answer any <u>two</u> of the following briefly: (4)

E 599-3

(i) Define a first countable space. In what sense the study of sequences is sufficient in first countable spaces?

(ii) Show that a filter $\mathscr{F}$ is an Ultrafilter on X if and only if for each subset E of X either E is in $\mathscr{F}$ or $X - E$ is in $\mathscr{F}$.

(iii) How do you generate a net out of a Filter?

(c) Answer all of the following very briefly: (3)

(i) Define a directed set giving genuine illustrations.

(ii) Define a neighbourhood of a point in a topological space. Show that the collection of all neighbourhoods of a a point in a topological space is a Filter.

(iii) How do you generate a Filter out of a Net?