Seat No. : _____

TD-116

M. Sc. Sem.-IV
May-2013
510 Mathematics
(Quantitative Techniques)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one :** **7**

   (i) Derive probability of x – successes for Poisson distribution using Bernoulli's trial.

   (ii) Find mean of negative binomial distribution.

   (b) Attempt any **one :** **4**

   (i) If $f(x)$ denotes the probability distribution of a random variable, x and $F(x)$ be its cumulative distribution function, prove that $E(x) = \int_0^\infty (1 - F(x))\,dx - \int_{-\infty}^0 F(x)dx$.

   (ii) For the rectangular population $dF = dx$, $0 \leq x \leq 2$, find $\mu_1'(0)$ and μ_2.

   (c) Choose the correct answer : **3**

   (i) If an event cannot take place, its probability will be

   (a) 1 (b) – 1

   (c) 0 (d) ∞

   (ii) If the events A and B are mutually exclusive, then P(A + B)

   (a) P(A) + P(B) (b) P(A) – P(B)

   (c) P(A) · P(B) (d) P(A)/P(B)

   (iii) If the events A and B are independent, the conditional probability of B given A, i.e. P(B/A) is

   (a) P(A) (b) P(B)

   (c) P(A) . P(B) (d) P(A$\cap$B)

2. (a) Attempt any **one :** **7**

(i) Derive an order – levels lot – size model when replenishment rate is infinite and shortages are allowed.

(ii) Discuss EOQ problem with inventory level constraint.

A small shop produces three machines parts A, B and C in lots. The shop has limited storage space sufficient only for 500 units of all type of items. The relevant data for the three items is given below :

Item	A	B	C
Demand rate (units / month)	600	1,200	1,500
Cost per unit (₹)	5	10	15
Set-up cost per lot (₹)	100	50	200

The inventory carrying charges for the shop are 20% of the average inventory valuation per month for each item. If stock-outs are not allowed, determine the optimum lot size for each item.

(b) Attempt any **one :** **4**

(i) What are the types of inventory ?

(ii) Define set-up cost, holding cost and shortage cost.

(c) Choose the correct answer : **3**

(i) If small orders are placed frequently, then total inventory cost is

(a) reduced (b) increased

(c) either reduced or increased (d) minimized

(ii) Economic order quantity (EOQ) results in

(a) equalization of holding cost and ordering cost

(b) minimization of set-up cost

(c) favourable ordering cost

(d) reduces chances of stock-outs

(iii) If the unit purchase cost increase, the optimum order quantity

(a) increases (b) decreases

(c) either increases or decreases (d) no change

3. (a) Attempt any **one :** **7**

(i) Derive difference – differential equations for ((M / M / 1) : (∞ / FCFS)) queue.

(ii) Derive average number of customers in ((M / M / 1) : (N / FCFS)) queue.

(b) Attempt any **one :** **4**

(i) Customers arrive at a sales counter manned by a single person according to a Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the waiting time of a customer.

(ii) Cars arrive at a petrol pump with exponential inter-arrival times having mean 12 minutes. The attendant takes an average of 1/5 minute per car to supply petrol, the service time being exponentially distributed. Determine (a) the average number of cars waiting to be served, (b) the average number of cars in the queue, and (c) the proportion of time for which the pump attendant is idle.

(c) Define any **three :** **3**

(i) Traffic intensity

(ii) Steady and transient state

(iii) Balking

(iv) Jockeying

(v) Queue discipline

4. (a) Attempt any **one :** **7**

(i) Discuss a model for the replacement of items whose maintenance costs increase with time and value of money remains same during the period.

(ii) Consider a group replacement model. There are N items in the group. Replacement is made after every t time periods. Assume that all the failures in a group are replaced at the end of the period. Further, C_1 is the cost of the replacing a unit in the group, and C_2 is the cost of replacing a failure with $C_2 > C_1$. Find an expression for the least cost associated with group replacement.

(b) Attempt any **one :** **4**

(i) The cost of a machine is ₹ 6,100 and its scrap value is ₹ 100. The maintenance costs found from experience are as follows :

Year	1	2	3	4	5	6	7	8
Main. cost (₹)	100	250	400	600	900	1200	1600	2000

When should the machine be replaced ?

(ii) A fleet owner finds from his past records what the costs per year of running a vehicle whose purchase price is ₹ 50,000 are as under :

Year	1	2	3	4	5	6	7
Running cost (₹)	5000	6000	7000	9000	11500	16000	18000
Resale value (₹)	30000	15000	9500	7500	2000	2000	2000

Thereafter, running cost increase by ₹ 2,000, but resale value remains constant at ₹ 2,000. At what age is the replacement due ?

(c) Choose the correct answer : **3**

(i) The problem of replacement is not about the

(a) items that deteriorate graphically

(b) items that fail suddenly

(c) determination of optimal replacement policy

(d) maintenance of an item to work out profitability

(ii) When time value of money is considered

(a) costs need to be discounted

(b) timing of incurrence of costs is important

(c) the present value factors serve as weights

(d) all of the above

(iii) Under group replacement policy

(a) group as well as individual replacements are done

(b) all the items are replaced, irrespective of the fact that items have failed or have not failed

(c) the optimal group replacement interval is determined at the point where the sum of group replacement per unit of time and the cost of individual replacement is maximum.

(d) All of the above

5. (a) Attempt any **one :** **7**

(i) The director of finance for a farm co-operative is concerned about yields per acre she can expect from this year's corn crop. The probability distribution of the yields for the current weather conditions is given below :

Yield in kg. per acre	Probability
120	0.18
140	0.26
160	0.44
180	0.12

She would like to see a simulation of the yields she might expect 10 years for weather conditions similar to those she is now experiencing :

(a) Simulate the average yield she might expect per acre using the following random numbers : 20 72 34 54 30 22 48 74 76 02

She is also interested in the effect of market price fluctuations on the cooperative's farm revenue. She makes this estimate of per kg. prices for corn :

Price / kg. (₹)	Probability
2.00	0.05
2.10	0.15
2.20	0.30
2.30	0.25
2.40	0.15
2.50	0.10

(b) Simulate the price she might expect to observe the next 10 years using the following random numbers : 82 95 18 96 20 84 56 11 52 03

(ii) A company trading motor vehicle spares wishes to determine the level of stock it should carry for the item in its range. Demand is not certain and replenishment of stock takes 3 days. For an item, the following information is obtained :

Demand (units per day)	1	2	3	4	5
Probability	0.1	0.2	0.3	0.30	0.10

Each time an order is placed, the company incurs an ordering cost of ₹ 20 per order. The company also incurs carrying cost of ₹ 2.50 per unit per day. The inventory carrying cost is calculated on the basis of average stock.

The manager of the company wishes to compare two options, for his inventory decision :

(i) Order 12 units when the inventory at the beginning of the day plus order standing is less than 12 units.

(ii) order 10 units when the inventory at the beginning of the day plus order standing is less than 10 units.

On first day, the company has a stock of 17 units.

The sequence of random numbers to be used is 08, 91, 25, 18, 40, 27, 85, 75, 32, 52 using first number for day one.

Carry out the simulation for 10 days and recommend which option is advantageous to the manager.

(b) Attempt any **one :** **4**

(i) What is simulation ? Give one application of simulation with details.

(ii) Outline the limitations of simulation ?

(c) Choose the correct answer : **3**

(i) The process of simulation

(a) is a powerful mathematical technique.

(b) is often referred to as "Monte-Carlo" simulation.

(c) usually requires use of computers to solve the problems.

(d) involve the criterion wherein the output of a simulation model is independent of the simulation run.

(ii) Analytic results are taken into consideration before a simulation study so as to

(a) determine the optimum solution

(b) identify suitable values of the system of parameters

(c) identify suitable values of decision variables for the specific choices of system parameters

(d) all of the above

(iii) One can increase the chance that results of simulation are not erroneous by

(a) validating the simulation model

(b) changing the input parameters

(c) using discrete probability distribution in place of continuous

(d) None of the above