Seat No. : _______________

XY-112

April-2013

M.Sc. (Sem.-II)

408 : Statistics

(Distribution Theory)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Describe the method of maximum likelihood to estimate the parameters of the Neyman type-A distribution.

OR

Define Neyman type-A distribution. Obtain its probability Generating function. Hence derive its r^{th} factorial cumulant. Also describe the method of fitting of Neyman type-A distribution to the numerical data.

(b) Define Poisson-Binomial distribution. Obtain its probability generating function. Show that Poisson-Binomial distribution tends to Poisson-Poisson distribution. State necessary assumptions involved.

OR

Let $x_1, x_2, ..., x_n$ are N identically independently distributed random variables and N is also a random variable independent of x_i's.

If $S_N = Y = \sum_{i=1}^{N} x_i$ then show that

(i) $E(S_N) = E(N)\, E(X)$

(ii) $V(S_N) = E(N) \cdot V(X) + V(N)\, \{EX)\}^2$

2. (a) Discuss the roll of non-central distributions in statistical inference with illustration. If $X \sim N(\mu, 1)$ then, obtain probability density function of non-central chi-square distribution using moment generating function.

OR

Define non-central 'F' distribution with (n_1, n_2) degrees of Freedom. In usual notations obtain probability density function of non-central 'F' distribution.

(b) Define non-central 't' statistic. In usual notations obtain probability density function of non-central 't' distribution.

OR

State and prove the relation between non-central chi-square, non-central F and non-central t distribution.

3. (a) Define Order Statistics. Obtain the distribution function of r^{th} order statistics. Also obtain the probability density function of r^{th} order statistics.

OR

Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population.

(b) If a random sample of size 'n' is taken from the exponential distribution with mean 1/3 then find the probability that the sample range does not exceed 3.

OR

If a random sample of size '4' is taken from uniform distribution U(0, 1) then derive the probability density function of the sample median.

4. (a) Define moments of order statistics. Show that $\mu_{r:n}$ exits if $E(1 \times 1 < \infty)$. In usual notations obtain the expressions for $\mu_{r:n}$ in terms of $F_r(x)$.

OR

Define rank-order statistics with appropriate example. Give functions definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

(b) Explain the procedure of obtaining confidence interval for p^{th} Quentile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cdf$^{F_x(x)}$ then obtain

$p(X_{(3)} <$ Population median $< X_{(5)})$.

OR

Obtain the correlation coefficient between r^{th} and s^{th} order statistics for the uniform distribution U(0, 1).

5. (a) Choose the correct answer :

(1) If $x_1, x_2, ..., x_n$ are independent variates each distributed as $N(\mu, \sigma^2)$ then the probability density function of $w = X_1 / \left(\frac{1}{n} \sum_{i=2}^{n} X_i^2\right)^{1/2}$ is

(a) 't' with n degree of freedom

(b) 't' with (n – 1) degrees of freedom

(c) Non-central 't' with n degrees of freedom

(d) None of these

(2) A non-central chi-square distribution is a

(a) Weighted sum of chi-square variables with weight as Poisson probabilities.

(b) Weighted sum of Poisson variables with weight as chi-square probabilities.

(c) Compound distribution of Poisson and chi-square distributions

(d) (a) and (c) but not (b)

(b) State whether the following statements are true or false. Justify your answer.

(1) For Poisson-Pascal distribution mean is less than variance.

(2) The correlation coefficient between the smallest and the largest order statistics for the uniform distribution U(0, 1) is 1/n.

(3) In probability generating function if we put '$z=(1-t)^{-1}$, then we get ascending factorial moment generation function.

(4) If $x_1, x_2, ..., x_n$ are n independent observations then the order statistics $y_1, y_2, ...y_n$ are independent order statistics.

(5) Rank-order statistics are invariant under monotone transformation.

(6) The i^{th} rank-order statistics $r(X_i)$ is the rank of the i^{th} observation in the original ordered sample.

(7) The sampling distribution of F-statistic does not involve any population parameter.

(c) Answer the following questions (in **one** sentence only)

(1) If a random sample of size '5' is taken from uniform distribution U(0, 1) then write the probability density function of the sample median.

(2) If X~N(μ, 1) and Y is an independent chi-square variate with n degrees of freedom then write the distribution $t = \frac{X}{\sqrt{Y/n}}$

(3) Write pgf of Poisson-Negative Binomial distribution.

(4) Write application of contagious distribution.

(5) Define descending factorial moment generating function.
