Seat No. : _______________

AE-118

April-2016

M.Sc., Sem.-IV

509-EA : Mathematics

(Mathematical Methods)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

(i) Find a basis of solutions of the differential equation $xy'' + 3y' + 4x^3y = 0$.

(ii) Using the indicated substitutions, reduce the following equation to Bessel's differential equation and find a general solution in terms of Bessel functions.

$x^2y'' + \frac{1}{4}\left(x + \frac{3}{4}\right)y = 0 \quad (y = u\sqrt{x}, \sqrt{x} = z)$

(b) Attempt any **two** : **4**

(i) Find a power series solution in powers of x of the equation $(x - 2)y' = xy$.

(ii) Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$.

(iii) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(c) Answer very briefly : **3**

(i) Why can the equation $y' = (y/x) + 1$ not be solved by a power series in powers of x ?

(ii) Find the indicial equation and its roots for the equation

$x^2y'' + x^3y' + (x^2 - 2)y = 0$

(iii) What is the value of $J_{1/2}(\pi)$?

2. (a) Attempt any **one** : **7**

(i) Find the inverse Laplace transform of $\frac{9}{s^2}\left(\frac{s+1}{s^2+9}\right)$.

(ii) Applying convolution, solve the initial value problem

$y'' + 2y' + 2y = 5u(t - 2\pi)\sin t;\ y(0) = 1,\ y'(0) = 0.$

(b) Attempt any **two** : **4**

(i) Using first shifting theorem, find the laplace transform of $5e^{2t} \sinh 2t$.

(ii) Find the Laplace transform of $te^{-t} \cos t$.

(iii) Find $L^{-1} \left\{ \frac{s}{(s^2 - 9)^2} \right\}$.

(c) Answer very briefly : **3**

(i) State the existence theorem for Laplace transform.

(ii) What is the Laplace transform of $\delta(t - \pi)$?

(iii) Write the function

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases}$$

in terms of unit step functions.

3. (a) Attempt any **one** : **7**

(i) Find the Fourier series of the function

$$f(x) = \begin{cases} k & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \ldots\ldots = \frac{\pi}{4}$.

(ii) Find the Fourier cosine and sine integrals of

$$f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

(b) Attempt any **two** : **4**

(i) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x^2 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(ii) Find the Fourier sine transform of e^{-x}.

(iii) Find the Fourier transform of

$$f(x) = \begin{cases} e^{-kx} & \text{if } x > 0 \ (k > 0) \\ 0 & \text{if } x < 0 \end{cases}$$

(c) Answer very briefly : **3**

(i) What is the fundamental period of $\cos 4x$?

(ii) Give an example of a function which is neither even nor odd. Justify your answer.

(iii) State Parseval's identity.

4. (a) Attempt any **one** : **7**

(i) Find the inverse Z-transform of $\frac{9z^3}{(3z-1)^2(z-2)}$ by residue method.

(ii) Solve the following difference equation by Z-transform.

$y_{k+3} - 3y_{k+2} + 3y_{k+1} - y_k = U(k),\ y_0 = y_1 = y_2 = 0,$

where

$$U(k) = \begin{cases} 0 & \text{if } k < 0 \\ 1 & \text{if } k \geq 0 \end{cases}$$

(b) Attempt any **two** : **4**

(i) Prove that if $Z[\{f(k)\}] = F(z)$, then $Z[\{a^k f(k)\}] = F\left(\frac{z}{a}\right)$.

(ii) Find the Z-transform of $\sin 2k,\ k \geq 0$.

(iii) Find the inverse Z-transform of $\frac{3z}{(z-2)}$ when $|z| < 2$.

(c) Answer very briefly : **3**

(i) State initial value theorem.

(ii) What is the order of the difference equation $6y_{k+2} - 2y_{k+1} + y_{k-1} = 0$?

(iii) What is the Z-transform of $f(k) = \frac{1}{3^k}$, $(-4 \leq k \leq 5)$?

5. (a) Attempt any **one** : **7**

(i) Prove that :

$$H_n\left\{\frac{df}{dx}\right\} = -s\left[\frac{n+1}{2n} H_{n-1}\{f(x)\} - \frac{n-1}{2n} H_{n+1}\{f(x)\}\right]$$

(ii) Show that :

$$\int_0^a x^3 J_0(sx)\, dx = \frac{a^2}{s^2}\left[2J_0(as) + \left(as - \frac{4}{as}\right) J_1(as)\right]$$

(b) Attempt any **two** : **4**

(i) Show that $H\{f(ax)\} = a^{-2}H\left(\frac{s}{a}\right)$.

(ii) Find the Hankel transform of $\frac{e^{-ax}}{x}$, n = 1

(iii) Find $H(x^n)$, $n > -1$ and $xJ_n(s_i x)$ is the kernel of the transform.

(c) Answer very briefly : **3**

(i) What is the value of the integral $\int_0^\infty \frac{e^{-ax}}{x} J_1(sx)\, dx$?

(ii) What is the value of the integral $\int_0^\infty x e^{-ax} J_0(sx)\, dx$?

(iii) Find $H^{-1}[e^{-as}]$, when n = 0.
