Seat No. : _______________

AE-102

April-2016

B.Sc., Sem.-VI

CC-310 : Mathematics

(Graph Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Figures to the right indicate full marks of the question/sub-question.

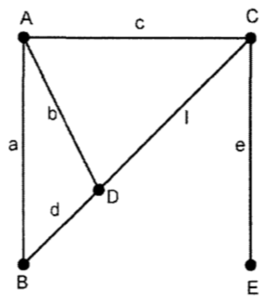
(3) Notations used in this question paper carry their usual meaning.

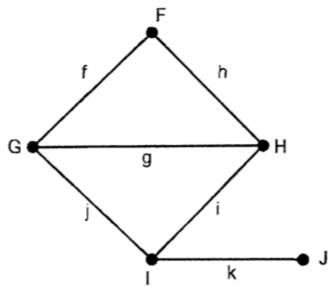
1. (a) If G is any graph with e edges and n vertices $v_1, v_2, v_3, ..., v_n$ then prove that **7**

$$\sum_{i=1}^{n} d(v_i) = 2e$$

OR

Define isomorphism of graphs. Show that the following graphs are isomorphic. **7**

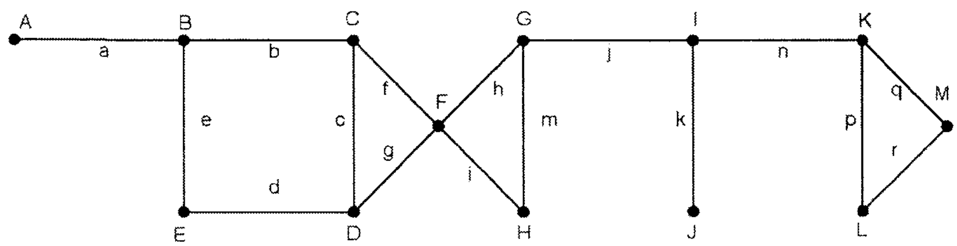




(b) Given any two vertices u, v $\in$ V(G), prove that every u – v walk contains a u – v path. **7**

OR

Find radius and diameter of the graph G **7**



2. (a) Draw all the trees (non-isomorphic) with number of vertices less than or equal to 5. **7**

OR

Prove that a non-empty graph G is bipartite if G has no odd cycles.

(b) Let u and v be any two distinct vertices of a tree T then prove that there is precisely one path from u to v. **7**

OR

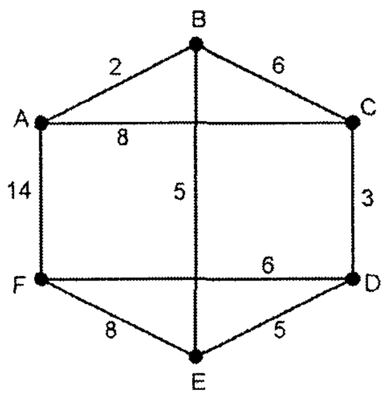
If T is a tree with n vertices then prove that it has precisely n – 1 edges.

3. (a) If G is a forest (acyclic graph) with n vertices and k connected components then prove that it has n – k edges. **7**

OR

If G is a connected graph then prove that it has a spanning tree.

(b) Apply the Dijkstra's algorithm on the following connected weighted graph to find the length of shortest paths from the vertex A to each of the other vertices of following graph. **7**



OR

Let G be a graph with n vertices $n \geq 2$ then prove that G has atleast two vertices which are not cut vertices.

4. (a) Write a short note on Konigsberg seven bridges problem. **7**

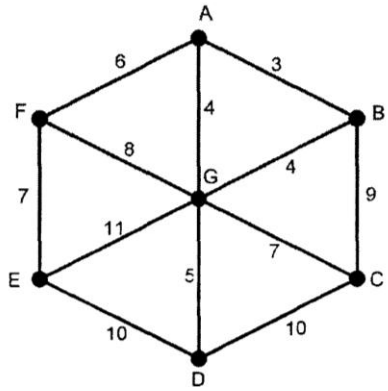
OR

A connected graph G is Euler if and only if the degree of every vertex is even.

(b) If G is simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, then prove that G is Hamiltonian. **7**

OR

Apply Prim's algorithm to find the minimal spanning tree on the graph :



5. Answer the following questions in short : (any **seven**) **14**

1. Define cycle and give an example.
2. Define k-regular graph and give an example.
3. Define trail with an example.
4. Find subgraph G – {B, E} for the graph G given in question 4 – (b).
5. Define Hamiltonian graph.
6. Find induced subgraph of G given in question 4 – (b), induced by U = {A, G, D}.
7. If connected graph G has 17 edges, what is the maximum possible number of vertices in G ?
8. Is the graph G with adjacency matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ connected ? Why ?
9. Define a complete graph with any one example of a complete graph.