Seat No. : _______________

AO-101

May-2016

B.Sc., Sem.-IV

CC-204 : Statistics
(Random Variable and Prob. Distribution – II)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : **All** questions carry equal marks.

1. (a) If $\phi(t) = e^{\lambda}(e^{it-1})$, then find $f(x)$. State the name of the distribution.

OR

(a) Let g(x) be a non-negative function and $f(x)$ be pmf or pdf of a random variable X, having finite expectation. Then prove that for k > 0,

$$P[g(X) \geq k] \leq \frac{E[g(X)]}{k} \text{ and } P[g(X) < k] \geq 1 - \frac{E[g(X)]}{k}$$

(b) State and prove the Jensen's inequality. Hence show that $E(X^2) \geq [E(X)]^2$.

OR

(b) State and prove Boole's inequality and Bonferroni's inequality.

2. (a) Define normal distribution. Derive its moment generating function. Hence find coefficient of skewness and kurtosis of normal distribution.

OR

(a) Define gamma distribution. Obtain expression for r^{th} raw moment of gamma distribution. Hence determine skewness and kurtosis of gamma distribution.

(b) Define Weibull distribution. Obtain the distribution function of Weibull distribution.

OR

(b) Let X ~ G(μ) and Y ~ G(ν). Both X and Y are independent. Obtain the distribution of U = X + Y and $Z = \frac{X}{X+Y}$.

3. (a) Define conditional expectation and conditional variance. With the usual notations prove that
   (i) $E[E(X|Y)] = E(X)$,
   (ii) $E[E(X^2|Y)] = E(X^2)$,
   (iii) $V(X) = E[V(X|Y)] + V[E(X|Y)]$.

OR

(a) For discrete bivariate distribution, define product moments about means. Define Product moment correlation coefficient in terms of product moments about means.

(b) Show that there is no value of k for which
$f(x, y) = ky\,(2y - x)$, $x = 0, 3$; $y = 0, 1, 2$
can serve as the joint probability distribution of two random variables.

OR

(b) Let the joint distribution of X and Y be
$f(x, y) = c(x^2 + y^2)$, $x = -1, 0, 1, 3$; $y = -1, 2, 3$
find (i) c, (ii) find the marginal distribution of X and marginal distribution of Y.

4. (a) With respect to Markov chain define (i) Reachable State, (ii) Transient State, (iii) Communicative State, (iv) Irreducible and Reducible State, (v) Recurrent State.

OR

(a) Explain Markov chain with stationary assumptions, also define transition probability matrix.

(b) Define stochastic process. Also explain discrete time and continuous time stochastic process with one example each.

OR

(b) Suppose that whether or not it rains tomorrow depends on previous weather conditions only through whether or not it is raining today. Suppose further that it it is raining today, then it will rain tomorrow with probability α, and if it is not raining today, then it will rain tomorrow with probability β. Define states for the given Markov Process and write the transition probability matrix.

5. Write answers in short :

(a) Let $X \sim W(\mu, \alpha, \beta)$. State the pdf and cdf of X.

(b) State moment generating function and cumulant generating function of gamma distribution.

(c) State the necessary conditions for a function to be a characteristic function.

(d) State the characteristic function of Poisson distribution.

(e) Define Absorbing State and Periodic State.

(f) State the Kendal's form of inversion theorem.

(g) Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$. Both are independently distribution. State the distribution of $\alpha_1 X_1 + \alpha_2 X_2$ and $\alpha_1 X_1 - \alpha_2 X_2$.
