Seat No. : _______________

AK-122

April-2016

M.Sc., Sem-II (CA-IT) Integrated

Matrix Algebra & Graph Theory

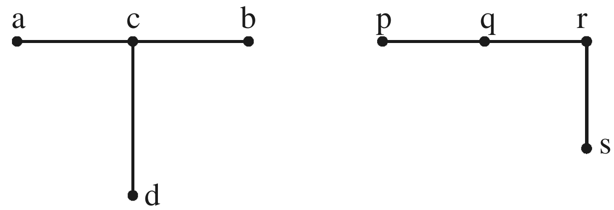
Time : 3 Hours] **[Max. Marks : 100**

1. Do as directed : (any **10**) **2 × 10 = 20**

   (i) Define : Complete graph, Isomorphic graph.

   (ii) Define : Bipartite graph, k-regular graph.

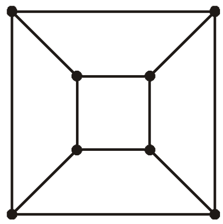
   (iii) Are they isomorphic :

   Justify your answer.

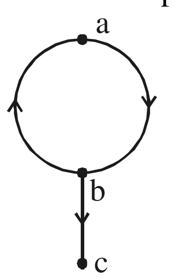
   (iv) Draw a complete graph with 5 vertices.

   (v) Is the following graph a complete bipartite graph ?

   Give reason

   (vi) Give the matrix representation of the graph.

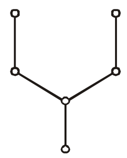
   

   (vii) Draw the graph for the given adjacency matrix :

   $$\begin{pmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

   (viii) Define Path, walk.

(ix) Find the square of the following graph :



(x) A tree has 3 vertices of degree 3 each, what is the number of leaves in the tree ?

(xi) Define : balanced tree of height h, Binary tree.

2. Answer any **two** : **10 × 2 = 20**

(a) Define :

(i) Cut vertex

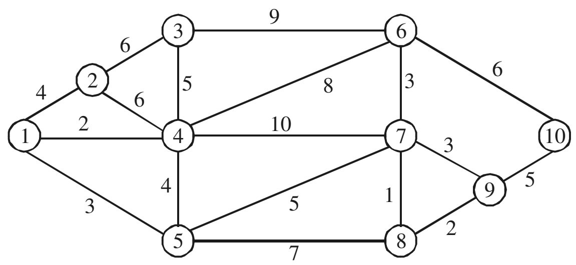
(ii) Bridge

(iii) Strongly connected graph

(iv) Spanning tree

(v) Directed graph

(b) Using Krushkal's Algorithm, find the minimal spanning tree from the following graph :



(c) Find the shortest path from the node 0 to node 8, from the data given below :

Arc (i-j)	Distance
0 – 1	16
0 – 2	20
0 – 3	45
1 – 3	19
2 – 4	13
2 – 5	14
3 – 5	11
3 – 7	21
4 – 5	9
5 – 6	13
5 – 7	18
6 – 7	12
6 – 8	10
7 – 8	15

(Use Dijkstra's algorithm)

3. Answer any **two** : **$10 \times 2 = 20$**

(a) (i) State Caley-Hamilton theorem. For the matrix given below, verify the theorem :

$$\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

(ii) Determine the Rank of the following matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

(b) (i) Test the consistency of the following set of equations :

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

(ii) Find the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(c) (i) Given v_1 and v_2 in a vector space V, let H = span $\{v_1, v_2\}$. Show that H is a subspace of V.

(ii) Define Linear Transformation. Let $T : R^3 \to R^2$ is given by $T(x, y, z) = (x + y, y + z)$. Is T a linear transformation ?

4. Answer any **four** : **$4 \times 5 = 20$**

(i) Find the inverse of the following matrix :

$$\begin{bmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{bmatrix}$$

(ii) Verify the theorem : det AB = det A debt B, where A, B are $n \times n$ matrices, for the following matrices :

$$A = \begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$$

Also verify if det (A + B) = det A + det B.

(iii) Compute det A, where

$$A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$$

(iv) Prove that the complete graph K_4 is plannar but K_5 is not.

(v) Define : (A) Jordan Curve, (B) Crossing number of a graph, (C) Hamiltonian graph, (D) Konisberg bridge problem, (E) Plannar graph.

5. (a) Answer any **five** : **5 × 3 = 15**

(i) How many matrices of order 2 × 3 can be formed, in which the digits from 0 to 9 occur not more than once ?

(ii) How many numbers are there between 1000 and 10000 in which all the digits are distinct ?

(iii) A hotel has six single rooms, six double rooms and four rooms where in each three persons can stay. In how many ways can 30 persons be accommodated in this hotel ?

(iv) X knows Tamil, Malayalam and Marathi. Y knows only Bengali and Punjabi. Z knows all these languages. In how many different ways can Z get a message from X and pass it onto Y ?

(v) Out of 200 students, 50 take Discrete Mathematics, 140 take Economics, 24 take both. How many of them do not take either of these courses ?

(vi) State Pigeon Hole principle.

(b) If A is a 7 × 9 matrix with a two dimensional null space, what is the rank of A ? Could 6 × 9 matrix have a two dimensional null space ? **5**

OR

The matrices given below are row equivalent :

$$A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 5 & -5 & -7 & 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -2 & -4 & 3 & -2 \\ 0 & 3 & 9 & -12 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(i) Find rank A and dim Nul A.

(ii) Find bases for Col A and Row A.

(iii) What is the next step to perform to find a basis for Nul A ?

(iv) How many pivot columns are in a row echelon form of A^T ?