Seat No. : _______________

AS-102

May-2016

B.C.A., Sem.-II

CC-111 : Mathematical Foundation of Computer Science

Time : 3 Hours] **[Max. Marks : 70**

1. (A) (1) Verify the properties, existence of identity and existence of inverse (for each of the elements of a set) for the following binary operations on a set of positive integers. **8**

(i) $a * b = a + b - 1$

(ii) $a * b = \frac{ab}{2}$

(2) Prove that every cyclic group is abelian.

OR

(A) (1) Show that a set $G = \{(a, b, c) / a, b, c \in R\}$ is a group with respect to the operation addition defined as follows : **8**

For any $\alpha = (a_1, b_1, c_1)$, $\beta = (a_2, b_2, c_2) \in G$, $\alpha + \beta = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(2) Prove that identity element in a Group is unique.

(B) (1) Show that a set $G = \{2^n / n \in Z\}$ under multiplication is a cyclic group. **6**

(2) Show that if G is an abelian Group, then for any $a, b \in G$, $(ab)^2 = a^2 b^2$.

OR

(1) Find the order of each element of a multiplicative group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$. **6**

(2) State Lagrange's Theorem. How many subgroups are there for the group of order 11 ?

2. (A) (1) Let a set $X = \{1, 2, 3, 4\}$. The relation matrix M(R) on a set X is given below : **8**

$$M(R) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Answer the following questions.

(i) Give Domain and Range of the relation.

(ii) Is this relation reflexive ?

(iii) Is this relation irreflexive ?

(iv) Is this relation Symmetric ?

(2) Define an equivalence relation and equivalence classes. Is the following relation an equivalence on the set $X = \{1, 2, 3\}$.

$R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 1)\}$

OR

(1) Give relation matrix and relation graph of a relation on a set X = {1, 2, 3, 4} defined by R = {⟨x, y⟩ / x, y ∈ X, x ≤ y}. **8**

(2) Show that a set ⟨P(A), ⊆⟩ , where A = {1, 2, 3}, with the relation inclusion is a Poset.

(B) (1) Draw the Hass diagram of the Poset $\langle S_{210}, D\rangle$. **6**

(2) Find minimal and maximal elements for the Poset ⟨P, D⟩, where P = {2, 3, 6, 12, 18, 36}, and D means "divides". Also find greatest lower bound of a subset (6, 12, 18} of P.

OR

(1) Define partition of a set. Determine whether or not the following sets are partition of the set Z of integers with justification. **6**

$p_l = \{\{x \in Z / x < 5\}, \{x \in Z / x > 5\}\}$

$p_2 = \{\{2n / n \in Z\}, \{2n + l / n \in Z\}\}$

(2) Define Chain. Give an example of a Poset which is not a chain.

3. (A) (1) Show that the operations of meet and join on a lattice are commutative and idempotent. **8**

(2) Define a Sub-Boolean Algebra. Find any three sub-Boolean Algebra of the Boolean Algebra $\langle S_{66}, D\rangle$ where S_m is a set of divisors of m and D is a partial ordering relation divides.

OR

(1) Define Boolean Algebra. Prove that in a Boolean Algebra, $a \leq b \Leftrightarrow b' \leq a'$ **8**

(2) Define Complete lattice. Is a lattices $\langle S_6, D\rangle$, complete ? Justify your answer. (S_m is a set of divisors of m and D is a partial ordering relation divides)

(B) (1) If exists, find complement of each element of a lattice ⟨P(A), ∩, ∪⟩, where A = {1, 2, 3}. **6**

(2) In a Boolean Algebra prove that, $a = b \Rightarrow ab' + a'b = 0$

OR

(1) By giving an example show that any subset of a lattice need not be a sublattice. **6**

(2) Let ⟨L, *, ⊕⟩ be a chain with L = {a, b, c}. If $a \leq b \leq c$, show that L is a distributive lattice.

4. (A) (1) Are the following graphs Isomorphic ? Justify your answer. **8**

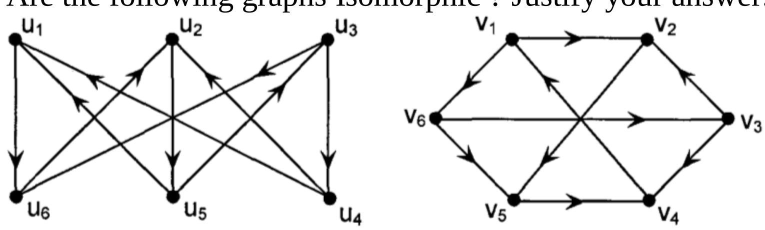

Figure – 1

(2) Give other representation of the Tree expressed by,

$(V_0(V_1(V_2)(V_3(V_4)))(V_5(V_6)(V_7(V_8)(V_9)))(V_{10}(V_{11}(V_{12}))))$

OR

(A) (1) Are the simple graphs with the following adjacency matrices isomorphic ? Draw the graph of each adjacency matrices. **8**

$$M(G_1) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } M(G_2) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(2) Define binary tree. Give the binary tree representation for the following tree representation.

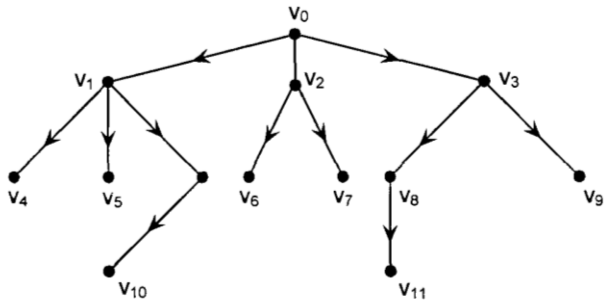

Figure – 2

(B) (1) Find the reachable set of vertices v_1, v_5 and v_{10} form the following graph. **6**

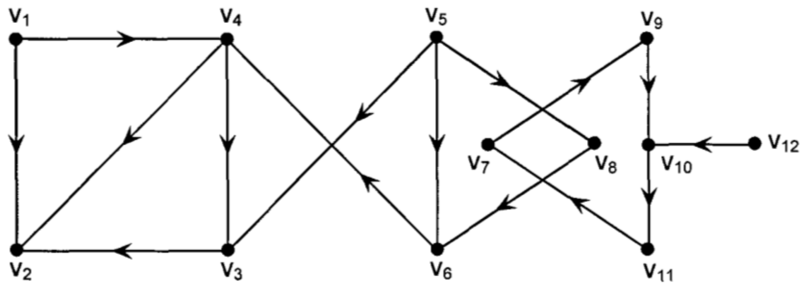

Figure – 3

(2) Define node base. Find the node base of the graph given in figure-3 above.

OR

(1) From the given graph G answer the following questions.

(i) Give a geodesic path from the vertex 'v_5' to the vertex 'v_1'.

(ii) Find the distance between two vertices v_1 and v_5.

(iii) Give the reachable set of a set $\{v_1, v_2, v_4\}$.

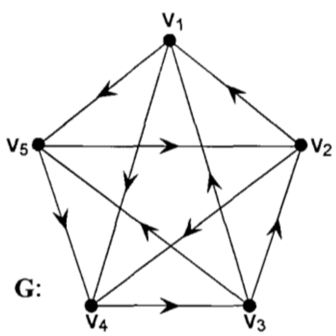

Figure - 4

(2) Give an adjacency matrix for the graph G given in the figure – 4 above. Also draw the subgraph H with $V(H) = V(G) - \{v_1\}$.

5. Do as Directed. **14**

(1) A cyclic group has only one generator. (True / False)

(2) The set N of all positive integers is a group with respect to operation ______.

(a) Addition (b) Multiplication

(c) $a * b = a + b - 2$ (d) None of these

(3) In the additive group of integers the order of every element is ______.

(a) Zero (b) n

(c) infinite (d) None of these

(4) Every subgroup of an abelian group is abelian. (True / False)

(5) There are ______ distinct permutations on a set of n elements.

(a) n (b) n + l

(c) n – 1 (d) None of these

(6) A covering of a set is always a partition of that set. (True / False)

(7) A relation is symmetric if its matrix is ______.

(a) Symmetric (b) Anti-symmetric

(c) Square (d) None of these

(8) If the Domain and Range of a relation are same then relation is ______.

(a) Reflexive (b) Symmetric

(c) Equivalence (d) None of these

(9) A Poset P is a lattice if for any $a, b \in P$, ______.

(a) $a * b \in P$ (b) $a * b = 0$

(c) $(a * b)' = a' \oplus b'$ (d) None of these

(10) Every Boolean Algebra is a lattice. (True / False)

(11) $\langle S_6, D\rangle$ is a sublattice of a lattice $\langle S_{12}, D\rangle$.

(a) $\langle S_{12}, D\rangle$ (b) $\langle S_{30}, D\rangle$

(c) $\langle S_{45}, D\rangle$ (d) None of these

(12) Every subset of a lattice is a sublattice. (True / False)

(13) Let a bounded lattice $\langle L, *, \oplus, 0, 1\rangle$. An element $b \in L$ is called a complement of an element $a \in L$ if ______.

(a) $a * b = a$ and $a \oplus b = b$ (b) $a * b = b * a$ and $a \oplus b = b \oplus a$

(c) $a * b = 0$ and $a \oplus b = 1$ (d) None of these

(14) In a Boolean Algebra $\langle B, *, \oplus, ', 0, 1\rangle$, for any $a, b \in B$, $(a * b)' =$ ______.

(a) a' (b) 0

(c) l (d) None of these
