**Seat No. : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

## AS-102

# May-2016

**B.C.A., Sem.-II**

**CC-111 : Mathematical Foundation of Computer Science**

**Time : 3 Hours] [Max. Marks : 70**

1. (A) (1) Verify the properties, existence of identity and existence of inverse (for each of the elements of a set) for the following binary operations on a set of positive integers. **8**

 (i) a \* b = a + b – 1

 (ii) a \* b =

 (2) Prove that every cyclic group is abelian.

**OR**

 (A) (1) Show that a set G = {(a, b, c) / a, b, c ∈ R} is a group with respect to the operation addition defined as follows : **8**

 For any  = (a1, b1, c1),  = (a2, b2, c2) ∈ G,  +  = (a1 + a2, b1 + b2,  c1 + c2)

 (2) Prove that identity element in a Group is unique.

 (B) (1) Show that a set G = {2n / n ∈ Z } under multiplication is a cyclic group. **6**

 (2) Show that if G is an abelian Group, then for any a, b ∈ G, (ab)2 = a2 b2.

**OR**

 (1) Find the order of each element of a multiplicative group G = {a, a2, a3, a4, a5, a6 = e}. **6**

 (2) State Lagrange’s Theorem. How many subgroups are there for the group of order 11 ?

2. (A) (1) Let a set X = {1, 2, 3, 4}. The relation matrix M(R) on a set X is given below : **8**

 M(R) =

 Answer the following questions.

 (i) Give Domain and Range of the relation.

 (ii) Is this relation reflexive ?

 (iii) Is this relation irreflexive ?

 (iv) Is this relation Symmetric ?

 (2) Define an equivalence relation and equivalence classes. Is the following relation an equivalence on the set X = {1, 2, 3}.

 R = {(1, 1), (1, 2), (1, 3), (2, 2), (3, 1)}

**OR**

 (1) Give relation matrix and relation graph of a relation on a set X = {1, 2, 3, 4} defined by R = {〈x, y〉 / x, y ∈ X, x < y}. **8**

 (2) Show that a set 〈P(A), ⊆) , where A = {1, 2, 3}, with the relation inclusion is a Poset.

 (B) (1) Draw the Hass diagram of the Poset 〈S210, D〉. **6**

 (2) Find minimal and maximal elements for the Poset 〈P, D〉, where  P = {2, 3, 6, 12, 18, 36}, and D means “divides”. Also find greatest lower bound of a subset (6, 12, 18} of P.

**OR**

 (1) Define partition of a set. Determine whether or not the following sets are partition of the set Z of integers with justification. **6**

 pl = {{x ∈ Z / *x* < 5}, {*x* ∈ Z / *x* > 5}}

 p2 = {{2n / n ∈ Z}, {2n + l / n ∈ Z}}

 (2) Define Chain. Give an example of a Poset which is not a chain.

3. (A) (1) Show that the operations of meet and join on a lattice are commutative and idempotent. **8**

 (2) Define a Sub-Boolean Algebra. Find any three sub-Boolean Algebra of the Boolean Algebra 〈S66, D〉 where Sm is a set of divisors of m and D is a partial ordering relation divides.

**OR**

 (1) Define Boolean Algebra. Prove that in a Boolean Algebra, a < b ⇔ b' < a' **8**

 (2) Define Complete lattice. Is a lattices 〈S6, D〉, complete ? Justify your answer. (Sm is a set of divisors of m and D is a partial ordering relation divides)

 (B) (1) If exists, find complement of each element of a lattice 〈P(A), ∩, ∪〉, where A = {1, 2, 3}. **6**

 (2) In a Boolean Algebra prove that, a = b ⇒ ab' + a'b = 0

**OR**

 (1) By giving an example show that any subset of a lattice need not be a sublattice. **6**

 (2) Let 〈L, \*, ⊕〉 be a chain with L = {a, b, c}. If a < b < c, show that L is a distributive lattice.

4. (A) (1) Are the following graphs Isomorphic ? Justify your answer. **8**



**Figure – 1**

 (2) Give other representation of the Tree expressed by,

 (V0(V1(V2)( V3(V4)))( V5(V6)( V7(V8)(V9)))(V10(V11(V12))))

**OR**

 (A) (1) Are the simple graphs with the following adjacency matrices isomorphic ? Draw the graph of each adjacency matrices. **8**

 M(G1) = and M(G2) =

 (2) Define binary tree. Give the binary tree representation for the following tree representation.



**Figure – 2**

 (B) (1) Find the reachable set of vertices v1, v5 and v10 form the following graph. **6**



**Figure – 3**

 (2) Define node base. Find the node base of the graph given in figure-3 above.

**OR**

 (1) From the given graph G answer the following questions.

 (i) Give a geodesic path from the vertex ‘v5’ to the vertex ‘v1’.

 (ii) Find the distance between two vertices v1 and v5.

 (iii) Give the reachable set of a set {v1, v2, v4}.



**Figure - 4**

 (2) Give an adjacency matrix for the graph G given in the figure – 4 above. Also draw the subgraph H with V(H) = V(G) – {v1}.

5. Do as Directed. **14**

 (1) A cyclic group has only one generator. (True / False)

 (2) The set N of all positive integers is a group with respect to operation \_\_\_\_\_.

 (a) Addition (b) Multiplication

 (c) a \* b = a + b – 2 (d) None of these

 (3) In the additive group of integers the order of every element is \_\_\_\_\_.

 (a) Zero (b) n

 (c) infinite (d) None of these

 (4) Every subgroup of an abelian group is abelian. (True / False)

 (5) There are \_\_\_\_\_ distinct permutations on a set of n elements.

 (a) n (b) n + l

 (c) n – 1 (d) None of these

 (6) A covering of a set is always a partition of that set. (True / False)

 (7) A relation is symmetric if its matrix is \_\_\_\_\_.

 (a) Symmetric (b) Anti-symmetric

 (c) Square (d) None of these

 (8) If the Domain and Range of a relation are same then relation is \_\_\_\_\_.

 (a) Reflexive (b) Symmetric

 (c) Equivalence (d) None of these

 (9) A Poset P is a lattice if for any a, b ∈ P, \_\_\_\_\_.

 (a) a \* b ∈ P (b) a \* b = 0

 (c) (a \* b)' = a' ⊕ b' (d) None of these

 (10) Every Boolean Algebra is a lattice. (True / False)

 (11) 〈S6, D〉 is a sublattice of a lattice 〈S12, D〉.

 (a) 〈S12, D〉 (b) 〈S30, D〉

 (c) 〈S45, D〉 (d) None of these

 (12) Every subset of a lattice is a sublattice. (True / False)

 (13) Let a bounded lattice 〈L, \* , ⊕, 0, 1〉. An element b ∈ L is called a complement of an element a ∈ L if \_\_\_\_\_.

 (a) a \* b = a and a ⊕ b = b (b) a \* b = b \* a and a ⊕ b = b ⊕ a

 (c) a \* b = 0 and a ⊕ b = 1 (d) None of these

 (14) In a Boolean Algebra 〈B, \*, ⊕, ‘, 0, 1〉, for any a, b ∈ B, (a \* b)' = \_\_\_\_\_.

 (a) a' (b) 0

 (c) l (d) None of these

\_\_\_\_\_\_\_\_\_\_\_\_\_