Seat No. : _______________

AR-129

May-2016

M.Sc., Sem.-II

408 : Statistics
(Distribution Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.

(2) Scientific calculators can be used.

1. (a) Define Neyman type A distribution. Obtain its probability generating function. Hence derive its r^{th} factorial cumulant. Also describe the method of fitting Neyman type A distribution to the numerical data.

OR

Let $X_1, X_2, ..., X_N$ are N identically independently distributed random variables and N is also a random variable independent of x_i's . Show that

(i) $E(S_N) = E(N)\,E(X)$

(ii) $V(S_N) = E(N)V(X) + V(N)\{E(X)\}^2,\ S_N = \sum_{i=1}^{N} X_i$

(b) Describe the method of maximum likelihood to estimate the parameters of the Poisson-poisson distribution.

OR

Define Poisson-Binomial distribution. Obtain its probability generating function. Show that Poisson-Binomial distribution tends to Poisson -Poisson distribution. State necessary assumptions involved.

2. (a) Define Poisson-Pascal distribution. Obtain recurrence relations for Probabilities and descending factorial moments for this distribution.

OR

Discuss the roll of non-central distributions in statistical inference. If X~ N(μ, 1) then, obtain probability density function of non-central Chi-square distribution using M.G.F.

(b) Define non-central ‘F’ distribution with (m, n_2) degrees of Freedom. In usual notations obtain probability density function of non-central ‘F’ distribution.

OR

Define non-central ‘t’ statistic. In usual notations obtain probability density function of non-central ‘t’ distribution.

3. (a) Obtain the joint probability density function of the largest and the smallest order Statistics.

OR

Let a random variable 'X' follows an Exponential distribution with mean θ, θ > 0. If a random sample of size n is taken from this distribution then show that $X_{(r)}$ and $X_{(S)} - X_{(r)}$ are independently distributed.

(b) Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population.

OR

Obtain the distribution of sample median when (i) n is odd number and (ii) n is even number.

4. (a) Prove that $(n - r)\,\mu^{(k)}_{r:n} + r\mu^{(k)}_{r+1:n} = n\mu^{(k)}_{r:n-1}$ where $\mu^{(k)}_{r:n}$ denotes k^{th} row moment of r^{th} order statistic from a random sample of size n.

OR

Explain the procedure of obtaining Confidence Interval for p^{th} Quantile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cumulative distribution function $F_x(x)$ then obtain $p\,(X_{(3)} < \text{Population median} < X_{(5)})$.

(b) Define rank-order statistics with appropriate example. Give functional definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

OR

Obtain the correlation coefficient between r^{th} and s^{th} order statistics for the uniform distribution U(0, 1). Hence write the correlation coefficient between the smallest and the largest order statistics.

5. Choose the correct answer.

(i) If $x_1, x_2, \ldots, x_m, x_{m+1}, \ldots, x_{m+n}$ are independent normal variates with zero mean and standard deviation σ then the distribution of $\sum_{i=1}^{m} x_i^2 \Big/ \sum_{i=m+1}^{m+n} x_i^2$ is

(a) F (m, n) (b) F(m, m + n)

(c) F'_λ (m, n) (d) None of these

(ii) If $x_1, x_2, \ldots, x_n$ are independent variates each distributed as $N(0, \sigma^2)$ then the probability density function of $w = x_1 \Big/ \left(\frac{1}{n}\sum_{i=1}^{m} x_i^2\right)^{1/2}$ is

(a) 't' with n degrees of freedom

(b) 't' with (n-1) degrees of freedom

(c) Non-central 't' with n degrees of freedom

(d) None of these

(iii) If a random variable X has a chi-square distribution with degrees of freedom 'r' and a random variable Y has a non-central chi-square distribution with degrees of freedom 1 and non-centrality parameter λ then the distribution of the random variable $Z = X + Y$ is

(a) Chi-square with degrees of freedom r + 1

(b) Non-central chi-square distribution with degrees of freedom r + 1 and non-centrality parameter λ

(c) Chi-square with degrees of freedom r

(d) None of these

(iv) A non-central chi-square distribution is a

(a) Weighted sum of chi-square variables with weight as Poisson probabilities

(b) Weighted sum of Poisson variables with weight as chi-square probabilities

(c) Compound distribution of Poisson and chi-square distributions

(d) (a) and (c) but not (b)

(v) The probability mass function of the Poisson Bionomial distribution is

(a) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{nr-x} \frac{\lambda^r}{r!}$

(b) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^{-x} q^{nr-x} \frac{\lambda^r}{r!}$

(c) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{-nr-x} \frac{\lambda^r}{r!}$

(d) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{nr}{x} p^x q^{nr-x} \frac{\lambda^{-r}}{r!}$

(vi) The probability generating function of the Poisson distribution is

(a) $G(Z) = e^{\lambda} + \lambda e^{-m+mz}$ (b) $G(Z) = e^{-\lambda} + \lambda e^{-m+mz}$

(c) $G(Z) = e^{\lambda} + \lambda e^{-m+mz}$ (d) $G(Z) = e^{-\lambda} + \lambda e^{-m+mz}$

(vii) The probability generating function of the Poisson Negative Binomial distribution is

(a) $G(Z) = e^{-\lambda - \lambda (q-pz)^{-n}}$ (b) $G(Z) = e^{\lambda - \lambda (q-pz)^{-n}}$

(c) $G(Z) = e^{-\lambda + \lambda (q-pz)^{-n}}$ (d) $G(Z) = e^{-\lambda + \lambda (q+pz)^{-n}}$

(viii) The recurrence relation for the probability of Neyman type-A distribution is

(a) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$ (b) $p_{r+1} = \frac{\mu_1' e^{-m}}{r-1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-j}$

(c) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j} p_{r-j}$ (d) $p_{r+1} = \frac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \frac{m^j}{j!} p_{r-1}$

(ix) Which one of the following statement is not true ?

(a) When 'v=l', student's t distribution tends to Weibull distribution.

(b) When 'v=l', student's t distribution tends to Cauchy distribution.

(c) The sampling distribution of F-statistic does not involve any population Parameter.

(d) The non-central Chi-square distribution is the mixture of central Chi-square distribution and Poisson distribution.

(x) Which one of the following statement is not true ?

(a) For Poisson Binomial distribution mean is less than variance.

(b) For Poisson Pascal distribution mean is less than variance.

(c) Neyman type-A distribution tends to Neyman type-B distribution.

(d) Neyman type-B distribution tends to Neyman type-A distribution.

(xi) The moment generating function of non-central chi-square distribution is

(a) $M\chi^2(t) = (1-2t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t < 1/2$

(b) $M\chi^2(t) = (1-2t)^{-1/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t > 1/2$

(c) $M\chi^2(t) = (1-2t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t > 1/2$

(d) $M\chi^2(t) = (1-2t)^{-n/2} \exp\left(\frac{2t\lambda}{1-2t}\right) \forall\, t \neq 1/2$

(xii) If X is a non-central chi-square variate with degree 5 and non-centrality parameter δ is also 5 then E(X) and V(X) are respectively

(a) (10, 30) (b) (15, 50)

(c) (5, 10) (d) None of these

(xiii) If a statistics t follows Student's t distribution with degrees of freedom n, then t^2 follows

(a) Student's t distribution with n^2 degrees of freedom

(b) Snedecor's F distribution with (1, n) degrees of freedom

(c) Snedecor's F distribution with (n, 1) degrees of freedom

(d) None of these

(ixv) Descending factorial cumulant generating function H(t) is defined as

(a) $\text{Log } E(l+t)^x$ (b) $\text{Ln } E(l+t)^x$

(c) $\text{Exp}(E(l+t)^x)$ (d) $\text{Log } E(l-t)^{-x}$

(xv) If a random sample of size 5 is taken from Uniform distribution then the probability density function of the sample median is

(a) the probability density function of the third order statistics

(b) the probability density function of the fifth order statistics

(c) the probability density function of the first order statistics

(d) None of these