

# STATISTICS

**GUJARAT UNIVERSITY**

**SYLLABUS OF**

**FIRST YEAR  
MASTER OF SCIENCE  
EXAMINATION**

**STATISTICS**

**(M.Sc. Part – I)**

# GUJARAT UNIVERSITY

## STATISTICS SYLLABUS AT M.A./M.Sc. (PARTS I & II)

**Part-I**      **Course effective from 1994-95**  
**Part-II**     **Course effective from 1995-96**

There will be four theory papers and three practical papers in each part. Each theory paper will be of 75 marks and of three hours' duration. Each practical paper will be of 35 marks and of three hours' duration. 45 marks are allotted for Viva Voce on theory papers covered at each part. In addition viva voce will include a submission of report on project on a topic on mathematical and applied statistics to be assigned to each student by the department. The report is to be prepared under the guidance of the teacher.

Minimum marks required for passing at Part-I and Part-II examination are 36 percent in (i) each of four theory papers, (ii) Practical papers (in aggregate) and (iii) Viva voce (inclusive of project). A candidate securing 48% or more but less than 60% will be placed in Second Class and a candidate securing 60% or more will be placed in First Class.

The structure of the M.A./M.Sc. Courses is as follows:

Part	Paper	Title	Workload in hours per work	Maximum marks	
I	Theory paper I	Matrix Algebra & Measure Theory	4 L	75	
	Theory paper II	Probability Theory & Distributions	4 L	75	
	Theory paper III	Theory of estimation and theory of sampling	4 L	75	
	Theory paper IV	Mathematical programming, Computer programming	4 L	75	
	Practical paper I	Based on relevant topics of theory papers I & II	3 hours	35	
	Practical paper II	Based on relevant topics of theory paper III	3 hours	35	
	Practical paper III	Based on relevant topics of theory paper IV	3 hours	35	
	Viva voce		2 hours	45	
	II	Theory paper V	Statistical inference	4 L	75
		Theory Paper VI	Design of experiments & Multivariate analysis	4 L	75
* Theory Paper VII		Optional paper	4 L	75	
* Theory Paper VIII		Optional paper	4 L	75	
Practical paper IV		Based on relevant topics of theory paper V	3 hours	35	
Practical paper V		Based on relevant topics of theory papers VI	3 hours	35	
Practical paper VI		Based on relevant topics of theory papers VII & VIII	3 hours	35	
Viva voce			2 hours	45	

\* Optional papers are to be selected from the list of Optional papers given below in consultation with the department of Statistics.

**M.A. / M.Sc. Part-I (Statistics)**  
(effective from 1994-95)

**Statistics Paper – I (Matrix Algebra & Measure Theory)**

**A. Matrix Algebra**

- (i) Matrices over of a field, operations on matrices, special types of matrices, trace and norm of a matrix.  
  
Determinants: Idea of permutation, theorems on determinants, Laplace method of expansion of a determinant. Properties of determinants.
- (ii) Rank of matrix – definition of row rank and column rank, properties of rank, Frobenius theorem, Homogeneous and non-homogenous linear equations and their consistency.
- (iii) Regular inverse of a matrix with properties, Inverse of partitioned matrices, generalized inverse of a matrix with properties, reflexive g-inverse and Moore-Penrose g-inverse
- (iv) Characteristic roots and vectors of a square matrix, properties of characteristic roots and characteristic vectors of symmetric, skew-symmetric, orthogonal normal, Unitary and Hermitian matrices.
- (v) Hermitian and quadratic forms – Classification of quadratic forms, reduction to diagonal forms. Simultaneous reduction of two quadratic forms, Maxima and Minima of the ratio of certain quadratic forms. Idempotent matrices and their properties, matrix version of Cochran's theorem.

**B. Measure Theory**

- (i) Algebra of sets – Classes of sets – field, ring, sigma field, sigma ring, generated field and ring, sequences of sets, limit superior and limit inferior and limit of sequence of sets. Borel sets in real line and p-dimensional Euclidean space.
- (ii) Measure on a field and sigma field, continuity property of a measure and continuity theorem on measure. Outer measure, construction of outer measure from measure. Caratheodory theorem on outer measure, outer measureability of sets, class of measurable sets. Extension theorem (without proof). Construction of Lebesgue measure on the real line, complete measure, Lebesgue-Stieltjes measure through distribution function, statement of correspondence theorem (without proof).
- (iii) Measurable function, algebra of measurable functions, composite measurable function, simple measurable and Borel-measurable functions. Sequences of measurable functions and measurability of their limit operations. Concepts of almost everywhere, almost uniform convergence, convergence in measure. Statement of Egoroy theorem (without proof).
- (iv) Integral of a measurable function with respect to a measure space and properties, indefinite integral, monotone, convergence theorem. Lebesgue-Stieltjes and Riemann-Stieltjes interability of a real valued function on a (finite) closed interval of the real line.
- (v) Definition of signed measure and properties, idea of absolute continuity and singularity of measure, statement of Radon-Nikodym theorem and applications. Measure on Cartesian product of measurable spaces, statements of product measure theorem and Fubini's theorem and examples.

## Statistics Paper – II (Probability Theory & Distributions)

### A. Probability Theory

- (i) Measure theoretic approach to probability – probability space, events, algebra of events, probability of occurrence of at least one, exactly and at least  $m$  events out of  $n$  events ( $1 < m < n$ ). Conditional probability, Bayes theorem and its applications independence of events.

Random variable as measurable function, distribution function of a random variable, decomposition of a distribution function, types of random variable – discrete, continuous and mixed. Joint distribution functions  $R^2$  and  $R^P$  with properties.

- (ii) Expectation, moments and conditional expectation with properties. Markov's, Chebyshev's, Holder's, Minkowski's and Jonson's inequalities, Liapounov's inequality and Berge's inequality.

Characteristic function and its properties; inversion theorem, convolution of distributions and convolution theorem (without proof), problem of momento.

- (iii) Sequence of random variables, independence of random variables, convergence in probability, in quadratic mean, in law of distribution and convergence almost surely and their interrelationship. Borel-Cantelli lemma Weak compactness theorem (Statement only). Helly-Bray theorem and continuity theorem due to Levy and Cramer and applications.
- (iv) Weak law of large numbers due to Chebyshev and Kinchine; Kolmogorov's inequality and statement of Hajek-Renyi inequality, Kolmogorov's strong law of large numbers (proof of sufficiency condition only). Central limit theorem (CLT); Statement of central limit law. CLT due to Lindeberg and Levy and Liapounov with proof. CLT due to Lindberg and Feller without proof and applications.
- (v) Elements of Stochastic Processes: Definition of Markov Chain and examples, Chapman – Kolmogorav's equation and  $n$ -step transition probabilities. Simple time dependent stochastic processes. Poison, pure birth processes and applications.

### B. Distributions

- (i) Power series distribution and properties, compound distribution and properties. Poisson-Poisson (neyman type A), Poisson-Binomial and Poisson-negative binomial distributions with properties.
- (ii) Non-central sampling distributions – non-central Chi-square,  $t$  &  $F$  distributions with properties.
- (iii) Multinomial and non-singular multivariate normal distributions with properties
- (iv) Quadratic forms in normal variables, Necessary and sufficient condition for a quadratic form to be distributed as Chi-square, independence of the two quadratic forms. Cochran's theorem and applications.
- (v) Order Statistics: Distribution (marginal and joint) of order statistics from continuous distributions and their properties. Coverage statistics and their distributions. Asymptotic normality of a sample quantile, joint normality of two sample quantiles.



## Statistics Paper-III (Theory of Estimation and Theory of Sampling)

### A. Theory of Estimation

- i) Formulation of basic problems of inference, point estimation – criteria of good estimators, consistency, unbiasedness, invariance and sufficiency, Minimum Variance Unbiased (MVU) estimators.  
Cramer-Rao's inequality and its importance, Fisher's information, MVE estimators, Chapman-Robbin's inequality and Bhattacharya bounds, generalization of Cramer-Rao inequality for multiparameter case (for regular distribution only).
- ii) Reduction of data through sufficient statistics, factorization criterion for sufficiency, direct derivation in sample cases, general proof for discrete (countable) case only. Minimal sufficiency definition, properties and illustrations, Exponential family of distributions – definition, properties and examples, definition of monotone likelihood ratio family. Rao-Blackwell theorem, completeness, Lehmann-Scheff theorem and UMVU estimators.
- iii) Methods of estimation – method of moments, method of maximum likelihood, minimum Chi-square and modified minimum Chi-square. Properties of ML estimators consistency, asymptotic normality, and asymptotic efficiency. ML estimation by method of scoring. Idea of APE. Definition of confidence interval, construction of confidence intervals using pivotal quantity. Examples from normal and exponential (regular case) distributions.
- iv) Linear estimation – estimability and best linear unbiased estimators (BLUE's), method of least squares and normal equations. Variances and covariances of squares estimators (LSE'S), Gauss-Markov theorem, estimation of error variance and examples.

### B. Theory of Sampling

- i) Review of basic concepts in sampling, sample selection and determination of sample size. Linear and circular systematic sampling, estimation of variance, population under linear trend.
- ii) Varying Probability Sampling : PPS sampling with replacement (WR estimation of population total and mean, its variance and variance estimators; efficiency with SRS – linear relationship, gain due to PPS sampling. Selection procedures- cumulative total methods, Lahiri's method.
- iii) Stratified PPS sampling and allocation of sample size. PPSWOR-Horwitz-Thompson (HT) estimator, its variance and variance estimators due to HT and YGS (Yates, Grund and Sen). Des Raj's estimator.
- iv) Cluster sampling – efficiency with SRS, varying clusters under SRS and PPS sampling.  
Two stage sampling – estimator of population mean and total under sampling with SRSWR, SRSWOR, PPSWR, optimum allocation for fixed cost and variance. Two phase sampling.
- v) Ratio estimator – its need, bias, MSE and variance, Ratio method of estimation under basic sampling schemes SRS, Systematic, PPSWR and stratified sampling, almost unbiased ratio estimator, unbiased ratio type estimators, double ratio estimator, product estimator.
- vi) Difference and regression estimator, regression method of estimation of bias and variance under SRS and stratified SRS.

Non sampling error's, Sources of non-sampling errors, nonresponse error.

## Statistics Paper – IV (Mathematical programming & computer programming)

### A. Mathematical programming

- i) General form of mathematical programming problem – Linear programming – Basic theorems – simplex method – Artificial variable technique – Charn's penalty method – Two phase method – Degeneracy – revised simplex method.
- ii) Duality in linear programming – Duality theorems – complementary slackness theorem – dual simplex method – sensitivity analysis in linear programming – parametric linear programming.
- iii) Applications of linear programming – Transportation and assignment problems – Game strategy – Two person zero sum problem into linear programming problem and vice versa.
- iv) Non-linear programming – constrained extremal problems – Lagrange's method – Kuhn-Tucker theory – Graphical and analytical solutions – Quadratic programming – Beal's and Wolfe's algorithms – separable, convex and fractional programming – elements of stochastic programming.
- v) Integer programming – Branch and Bound method – cutting plane algorithm – zero-one linear programming – Elements of dynamic programming with sample illustrative samples.

### B. Computer Programming

- i) Introduction to programming language : major high level languages, choosing programming language for statistical computations, characteristics of statistical method problems, programme design, featured of a good program documentation.
- ii) Programming with FORTRAN 77 : Data types: expressions and statement, input/output commands, conditional and interactive constraints, array manipulation.
- iii) Subprograms: Data sharing among subprograms/programs, file manipulation facilities.  
Various programs using above features.

**Practical paper I**                      Based on relevant topics of theory paper I & II

**Practical paper II**                      Based on relevant topics of theory paper III

**Practical Paper III**                      Based on relevant topics of theory paper IV

**Viva voce**

## List of the books

### Theory paper-I

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|-----|--------------------------------------|---|---------------------------------------|
| 1.  | C.G. Khatri                          | Mathematics of Matrix (in Gujarati)               | Granth Nirman Board, Ahmedabad (1971) |
| 2.  | S.R.                                 | Matrix Algebra useful for Statistics              | John Wiley (1982)                     |
| 3.  | Rao C.R.                             | Linear Statistical inference and its applications | Wiley Eastern (1973)                  |
| 4.  | Rao C.R. & Mitra S.K                 | Generalised inverses of matrices and applications |                                       |
| 5.  | Ramchandran Rao A., Bhimasankaran P. | Linear Algebra                                    | Tata Mcgraw Hill (1992)               |
| 6.  | G.                                   | Linear Algebra Andison                            | Wesley (1961)                         |
| 7.  | Shantirayan                          | A text book of matrix Algebra                     | S. Chand & Co., New Delhi.            |
| 8.  |                                      | Measure theory                                    | Van Nostrand                          |
| 9.  | Hangman, J.F.O. & Taylor S.J.        | Introduction to measure and probability           | Cambridge University Press (1966)     |
| 10. | Loeve M.                             | Probability theory                                | Van Nostrand (1963)                   |
| 11. | Barra B.                             | Measure Theory and Integration                    | (1981)                                |
| 12. | Rao MM                               | Measure Theory and Integration                    | (1988):                               |
| 13. | Burril C.W.                          | Measure Theory & Probability                      | Tata Mcgraw Hill (1972)               |

### Theory paper-II

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|-----|--------------------------|---|--------------------------------------|
| 1.  | Bhat P.R.                | Modern Probability Theory – An Introductory Text Book – 2 <sup>nd</sup> ed. | Wiley Eastern (1984)                 |
| 2.  | Loeve M.                 | Probability Theory, 3 <sup>rd</sup> ed.                                     | Van Nostrand (1963):.                |
| 3.  | Roller W.                | An Introduction to Probability Theory Vol. I                                | Wiley Eastern (1972)                 |
| 4.  | Parzon E.                | Modern Probability Theory and its Applications                              | John Wiley (1962)                    |
| 5.  | Martin E.                | Introduction to Mathematical Probability Theory                             | Prentice-Hall (1970)                 |
| 6.  | Goon, Gupta and Dasgupta | An outline of Statistical Theory Vol.I,                                     | World Press, Calcutta (1970)         |
| 7.  | Rohatgi, V.K.            | An introduction to Probability Theory                                       | Prentice-Hall (1984)                 |
| 8.  | Rao C.R.                 | Linear Statistical Inference and its Applications                           | Wiley Eastern (1973)                 |
| 9.  | Gibbons, J.D.            | Non Parametric Statistical Inferences                                       | Mcgraw Hill (1971)                   |
| 10. | Johnson & Kots           | Distribution in Statistics Vol. I & II                                      | John Wiley (1972)                    |
| 11. | Jaiswal, M.C.            | Statistical Distributions (in Gujarati)                                     | Gujarat University, Ahmedabad (1972) |
| 12. | Partha Sarthi, K.R.      | Introduction to Probability & Measure                                       | McMillan Co (1970)                   |
| 13. | Cacoullos T.             | Exercise in Probability   | Narosa (1989).                       |

### Theory Paper III

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|---|---|--|
| 1. Lehmann, E.L                                 | Theory of point estimation                                    | Wiley Eastern (1983)                                     |
| 2. Zacks, S.                                    | Theory of statistical inference                               | John Wiley (1971)  |
| 3. Rao C.R.                                     | Linear statistical inference and its applications             | Wiley Eastern (1973)                                     |
| 4. Dudewicz E. & Mishra, S.N.                   | Modern Mathematical Statistics                                | Wiley Eastern (1988)                                     |
| 5. Rohatgi, V.K.                                | Introduction to probability Theory & Mathematical Statistics  | John Wiley (1984)  |
| 6. Deutech, R.                                  | Estimation theory   | Prentice-Hall (1984)                                     |
| 7. Goon Gupta and Dasgupta                      | An outline of Statistical theory, Vol.II                      | World Press, Calcutta (1970)                             |
| 8. Wilks, S.S.                                  | Mathematical Statistics                                       | John Wiley (1962)  |
| 9. Murthy, M.N.                                 | Sampling theory and methods                                   | Statistical Pub., Soc., Calcutta (1967)                  |
| 10. Des Raj                                     | Sampling theory   | Tata Mcgraw Hill (1976)                                  |
| 11. Sukhatme, V. et.al.                         | Sampling theory of surveys with applications                  | Indian Soc. Of Agricultural Statistics, New Delhi (1984) |
| 12. Shah  | Survey sampling   | John Wiley (1984)  |
| 13. Hansen M.N. and Hurwitz, W.N. & Madow, W.G. | Sample survey methods and theory and applications Vol.I & II. | John Wiley (1953)  |
| 14. Cochran, W.S.                               | Sampling Techniques, 3 <sup>rd</sup> ed                       | Wiley Eastern (1984)                                     |

### Theory Paper IV

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|----------------------------------|---------------------------------------|----------------------------------|
| 1. Hadley G.S.                   | Linear programming                    | Addison-Wesley Co (1959)         |
| 2.                               | Non-linear & Dynamic Programming      | Addison-Wesley Co. (1964)        |
| 3. Gass, S.I.                    | Linear Programming                    | (1975)                           |
| 4. Taha, H.A.                    | O.R. – An introduction                | Mac Millan & Co. (1989)          |
| 5. Hillier, P.S. & Leberman G.S. | Introduction to O.R.                  | Holoen Day (1952)                |
| 6. J.R. Sharma                   | Operations Research                   | Tata Mcgraw Hill (1956)          |
| 7. Sharma S.D.                   | Operations Research                   | Kedar Nath-Ram Nath & Co. (1989) |
| 8. Kantiswarup & Gupta, M.M.     | O.R. S.                               | Chand & Co. (1965)               |
| 9. Rao S.S.                      | Optimisation theory & applications    | Wiley Eastern (1978)             |
| 10. Hu T.C.                      | Integer programming and Network flows | (1969)                           |
| 11. Rajaraman                    | Computer programming in Fortran 77    |                                  |
| 12. Ram Kumar                    | Programming with Fortran 77           | Tata Mcgraw Hill (1986)          |
| 13. Shelly John                  | Essentials of Fortran 77              | John Wiley                       |
| 14. Katzan P                     | Fortran 77                            |                                  |