Seat No. : _______________

NF-102

December-2015

B.Sc., Sem.-V

Elective-305 : Mathematics
(Number Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Notations are usual, everywhere.

1. (A) State and prove Division algorithm theorem.

OR

Prove that the linear Diophantine equation $ax + by = c$ has a solution iff d/c, where d = g.c.d (a,b). Also prove that if x_0, y_0 is a solution of this equation then all other solutions are given by

$x = x_0 + \left(\frac{b}{d}\right)t$; $y = y_0 - \left(\frac{a}{d}\right)t$ where t is any integer. **8**

(B) Attempt any **two** : **10**

(1) Define g.c.d for two integer, using the Euclidean algorithm to obtain integer x and y such that it satisfying g.c.d (2378, 1769) = 1769 x + 2378 y. Also find g.c.d. for 1769 and 2378.

(2) Find the all positive solutions in the integers for the Diophantine equation, $24x + 138y = 18$.

(3) If a and b are odd integers then prove that $16 \mid a^4 + b^4 - 2$.

2. (A) Define "congruent modulo n relation for a fixed positive integer n". Also prove that it is an equivalence relation.

OR

Define prime number. Also if p is a prime and p | ab | then prove that either p | a | or p | b |. **8**

(B) Attempt any **two** : **10**

(1) Using Chinese Remainder Theorem, solve :

$2x \equiv 1 \pmod 3$, $3x \equiv 1 \pmod 5$ and $5x \equiv 1 \pmod 7$.

(2) Assuming that 495 divides $273 \times 49\, y^5$, obtain the digits x and y.

(3) Prove that 5 is a factor of $3^{3n+1} + 2^{n+1}$ for each positive n.

3. (A) State and prove Euler's theorem.

OR

State and prove Wilson's theorem. **8**

(B) Attempt any **two** : **10**

(1) In usual notation prove that $2^{20} \equiv 1 \pmod{41}$

(2) Find the remainder when the sum $1! + 2! + 3! + \ldots + 100!$ is divisible by 12.

(3) Solve : $18\, x \equiv 30 \pmod{42}$.

4. Answer the following in short : **16**

(1) Define Euler's Phi function and find the value of $\phi\,(2013)$.

(2) Prove that the square of any odd integer is of the form $8k + 1$, where k is any integer.

(3) State (only) the Fermat's theorem.

(4) State the Fundamental theorem of arithmetic.

(5) Define Well-Ordering Principle.

(6) Prove that $a \equiv b \pmod n$ if and only if a and b leave the same non-negative remainder when divided by n.

(7) What is the necessary and sufficient condition for the linear congruent equation $ax \equiv b \pmod n$ has a solution ?

(8) For $a \geq 1$, Prove that $\frac{a(a^2 + 2)}{3}$ is an integer.

Seat No. : _______________

NF-102

December-2015

B.Sc., Sem.-V

Elective-305 : Mathematics (Discrete Mathematics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Figures to the right indicate full marks of the question/sub-question.

(3) Notations used in this question paper carry their usual meaning.

1. (a) Show that $(P(X), \subseteq)$ is a poset. Is it a chain ? **9**

OR

Let $(L, \leq)$ be a lattice. For any a, b, c $\in$ L, prove that

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

(b) For a lattice $(L, \leq)$ prove that **9**

$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

OR

Explain Hasse diagram and draw the Hasse diagram of (S_{150}, D).

2. (a) Prove that the direct product of any two distributive lattices is a distributive lattice. **9**

OR

Let $(L, \leq)$ be a lattice. For any a, b, c $\in$ L, prove that

$a \leq c \Rightarrow a \oplus (b * c) \leq (a \oplus b) * c$.

(b) State and prove De' Morgan's laws in a Boolean algebra. **9**

OR

Show that in a Boolean algebra

$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$.

3. (a) Let (B, $*$, $\oplus$, ', 0, 1) be a Boolean algebra. A be the set of all atoms of B and $x_1, x_2 \in$ B. Prove that. (any three) **9**

(1) $A(0) = \phi$

(2) $A(1) = A$

(3) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(4) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(5) $A(x') = A - A(x)$

OR

Prove that the sum of all minterms in n variables is 1.

(b) Find POS and SOP canonical forms of the Boolean expression **9**

$\alpha(x_1, x_2, x_3) = (x_1 \oplus x_2) * x_3$

OR

Let (L, $*$, $\oplus$) be a distributive lattice, for a, b, c $\in$ L, prove that

$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$.

4. Answer in short. **16**

(a) Define Boolean expression.

(b) Define equivalence relation.

(c) Define irreflexive relation.

(d) Draw the Hasse diagram of $S_2 \times S_3$.

(e) Draw the Hasse diagram of (S_{1001}, D).

(f) Find all atoms of (S_{30}, D) Boolean algebra.

(g) Is (S_9, D) a Boolean algebra ? Why ?

(h) Show that 0 is the only complement of 1 in a lattice.
