Seat No. : _____

AP-120

May-2016

M.Sc., Sem.-II

407 : Statistics
(Reliability and Life Testing and Bayes Estimation)

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (1) **All** the questions are of equal marks.
(2) Scientific calculator is allowed.

1. (a) In usual notations prove that $f(t) = h(t)\exp\{-\int_0^t h(x)dx\}$.

If $h(t) = \frac{\frac{1}{2} + \frac{1}{3}e^{t/6}}{1 + e^{t/6}}$, $t > 0$ obtain corresponding lifetime distribution.

OR

If m is mean time between failure (mtbf) and R(t) is reliability function then show that $m = \int_0^\infty R(t)dt$. If $R(t) = \frac{1}{2}e^{-t/2} + \frac{1}{2}e^{-t/3}$, $t > 0$ then obtain mtbf.

(b) Discuss nonparametric method to estimate R(t) and h(t) in case of grouped failure data.

OR

Let $X_1, X_2, ..., X_n$ are iid exponential variates with mean $\theta > 0$. Let $X_{(k)}$ be the k-th order statistic, k = 1, 2, ..., n and $G_k = X_{(k)} - X_{(k-1)}$ then obtain distribution of G_k and $E(X_{(k)})$.

2. (a) Why censoring is used in life testing experiment ? Discuss Type-I censoring with illustration. Obtain general form of the likelihood function under Type-I censoring without replacement(wor).

OR

Under Type-II censoring wor obtain mle of mean life time θ of exponential distribution. Show that it is unbiased. Obtain its variance.

(b) Obtain UMVUE of R(t) in case of Type-II censoring wor for exponential life time model with mean $\theta > 0$. Obtain variance of the UMVUE.

OR

Obtain mle of mean life time θ of exponential distribution under Type-I censoring wor. Show that it is biased estimator. Obtain its relative bias.

3. (a) Discuss estimation of the parameters of Weibull lifetime model under Type-I censoring wor.

OR

Discuss estimation of the parameters of Weibull lifetime model based on the estimate of failure rate given by Fisher.

(b) Show that the expected life of the series system of 2 independent components is less than the expected life of either component. Also show that it is reverse in case of parallel system having the same two components.

OR

Show the parallel series system would be always provide higher reliability than that of series-parallel system.

4. (a) Discuss estimation in Rayleigh life time model under Type-II censoring wor.

OR

What is Bayes estimation ? Discuss normal and extensive form of obtaining Bayes estimator.

(b) Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution with mean $\lambda > 0$ and λ follows exponential distribution with mean $\beta > 0$. Obtain Bayes estimator of (i) $1/\lambda$, (ii) e^{λ} under (1) squared error loss function (2) weighted squared error loss function with weight $w(\lambda) = \lambda^2$.

OR

Let $X_1, X_2, ..., X_n$ be a random sample from normal $N(\mu, \sigma^2)$ distribution, σ is known and μ follows normal $N(\delta, 1)$ distribution. Obtain Bayes estimator of μ under squared error loss function. Also obtain its Bayes risk.

5. Answer the following :

(i) Define reliability.

(ii) Define hazard function.

(iii) State reliability of a series system.

(iv) State reliability of a parallel system.

(v) Define mean time to failure.

(vi) Which distribution posses any types of hazard rate ?

(vii) State the formula for estimating R(t) by nonparametric method for ungrouped data.

(viii) State memoryless property.

(ix) Identify correct answer : If X is continuous non negative random variable with cumulative hazard rate H, the H follows :

(a) Binomial distribution

(b) Poisson distribution

(c) Standard exponential distribution

(d) Uniform distribution

(x) Identify correct answer : Let X(1), X(2), ..., X(n) are ordered statistics of a random sample of size n from exponential distribution with mean θ. Define $Z_i = (n - i + 1)(X_{(i)} - X_{(i-1)})$, $i = 1, 2, ..., n$; $X_{(0)} = 0$. Then the distribution of Z_i will be

(a) exponential with mean $n\theta$

(b) exponential with mean θ

(c) exponential with mean $1/\theta$

(d) exponential with mean $1 / n\theta$

(xi) State the distribution of number of failure in [0, t) when lifetime model is exponential and test is performed with replacement with n items on the test.

(xii) State expected termination time under exponential lifetime model under Type-II censoring wor.

(xiii) Define posterior distribution.

(xiv) Define Bayes estimator.