Seat No. : _______________

AP-119

May-2016

M.Sc., Sem.-II

407 :Mathematics
(Differential Geometry – I)

Time : 3 Hours] **[Max. Marks : 70**

1. (A) Find parametrizations of the following level curves : **7**

   (i) $x^2 - y^2 = 1$

   (ii) $x^2 + y^2 + 2x - 2y + 1 = 0$

   OR

   (A) Find the Cartesian equations of the following parametrized curves : **7**

   (i) $r(t) = (\cos^2(t), \sin^2(t))$,

   (ii) $r(t) = (e^t, t^4 + 1)$.

   (B) Answer any **two** : **4**

   (i) Let $r(t) = (t, \cosh(t))$. Calculate the arc length starting at the point (0, 1).

   (ii) Is the curve r given below unit speed ?

   $r(t) = \left(\frac{3}{5}\cos(t), -\sin(t), \frac{4}{5}\cos(t)\right)$.

   (iii) Let $r(t) = (t, t^2, t^3)$. Let α be the angle between r (1) and the tangent vector at r (1). Show that $\alpha \neq \frac{\pi}{2}$.

   (C) Answer **all**. **3**

   (i) Sketch the curve

   $r(t) = (\cos(t), \sin(t))$

   (ii) Sketch the curve

   $r(t) = (e^t \cos(t), e^t \sin(t))$

   (iii) Sketch the curve

   $r(t) = (t, \sin(t))$

2. (A) Compute k, c, t, n, b for the curve **7**

$r(t) = \left(\frac{4}{5}\cos(t), \frac{4}{5}\sin(t), \frac{3}{5}t\right)$.

Verify the Frenet – Serret equations.

OR

(A) Find the curvature and torsion for the curve **7**

$r(t) = (t, t^2, t^3)$.

Show that it is not a planar curve.

(B) Answer any **two** : **4**

(i) Suppose r (s) is a unit speed curve in $\mathbb{R}^2$.

Define its signed curvature.

(ii) Suppose r (t) = (t, sin (t)).

Find the signed curvature of r at the point (0, 0)

(iii) Is the curve given below planar ?

$r(t) = (1 + t^2, 1 + 2t + t^2, 1 + t)$.

(C) Answer **all**. **3**

(i) Give an example of a curve whose curvature is zero at every point. (Do not prove)

(ii) Give an example of a curve whose curvature is 1 at every point. (Do not prove)

(iii) Write down (without proof) a formula for the curvature of r (t).

3. (A) Show that the level surface. **7**

$\frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{9} = 1$

Is a smooth surface.

Find the equation of the tangent plane of this surface at the point (2, 0, 0).

OR

(A) Show that

$\sigma(r, \theta) = (r \cos h(\theta), r \sin h(\theta), r^2)$

Is a parametrization of the part z > 0 of the hyperbolic paraboloid $z = x^2 - y^2$.

Find the equation of the tangent plane at the point (1, 0, 1).

(B) Answer any **two** : **4**

(i) Show that an open disc in the xy – plane is a regular surface.

(ii) Show that the plane $x + y - z = 1$ can be covered by a single surface patch.

(iii) Give a tangent vector to the surface $x + y - z = 1$ at the point (1, 2, 3).

(C) Answer all. **3**

(i) Give (without proof) a parametrization $\sigma(u, v)$ of the unit sphere $x^2 + y^2 + z^2 = 1$.

(ii) Show that the line $r(t) = (t, -t, 0)$ is contained in the surface $\sigma(u, v) = (u, v, u^2 - v^2)$.

(iii) Give (without proof) an example of a closed bounded surface.

4. (A) Define a generalized cylinder S. Give a parametrization $\sigma(u, v)$ for S. Find necessary conditions for σ to be regular. Give an example of a generalized cylinder S. **7**

OR

(A) Define a surface of revolution. Show that the surface $x^2 + y^2 - z^2 = 1$. Is a surface of revolution and find a parametrization for this surface.

(B) Answer any **two** : **4**

(i) Which kind of quadric is S, where S is given by the equation ?

$$\frac{(z-1)^2}{2^2} - \frac{(x+1)^2}{3^2} - \frac{y^2}{5} = 1 \quad ?$$

(ii) Which kind of quadric is S, where S is given by the equation

$x^2 + y^2 + z^2 + 4x - 4y + 2z = 0$?

(iii) Define a triply orthogonal system (of surfaces).

(C) Answer all. **3**

(i) Show that the planes $x = 1$, $y = 2$, $z = 3$ intersect mutually perpendicularly at the point (1, 2, 3).

(ii) Give an example (without proof) of a triply orthogonal system.

(iii) Name the quadric given by $x^2 + y^2 - z^2 = 0$.

5. (A) Determine the area of the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$. **7**

OR

(A) Determine the area of the part of the unit sphere with latitude θ greater than $\frac{\pi}{6}$

(B) Answer any **two** : **4**

(i) Calculate the first fundamental form of the surface $\sigma(u, v) = (u, v, u^2 - v^2)$.

(ii) Calculate the first fundamental form of the surface

$\sigma(u, v) = (u - v, u + v, 2u + 3v)$.

(iii) Let $S = \{(x, y, o) \mid x, y \in \mathbb{R}\}$. Give an example of an equiareal map from the plane S to itself which is not an isometry. (Do not prove)

(C) Answer **all**. **3**

Consider the plane

$S = \{(x, y, o) \mid x, y \in \mathbb{R}\}$.

(i) Give an isometry of S to itself which is not the identity. (Do not prove)

(ii) Give a conformal map from S to itself which is not an isometry. (Do not prove)

(iii) Is the map $f(x, y, o) = (x, y + \sin(x), o)$ an onto map ?