Seat No. : _______________

AP-117

May-2016

M.Sc., Sem.-II

407 : Physics
(Quantum Mechanics-II & Mathematical Physics-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Symbols and terminology used have their usual meanings.
(2) Scientific calculator should allow if necessary.
(3) Assume suitable data wherever necessary.

1. (A) What is basic difference between Schrodinger picture and Heisenberg picture ? Show that in Heisenberg approach, equation of motions is Hamiltonian equation of motion. **7**

OR

State the limitation of Hartree approximation. Explain how such limitations are overcome by Hartree-Fock approximation.

Write modified Schrodinger equation in the form of abstract operator $\hat{F}$.

Derive necessary equation for $F(\vec{r})\ \varphi\ (r)$ in Hartree-Fock method.

(B) Discuss Thomas-Fermi approximation for electron gas. **7**

OR

Show that $n(r) = \frac{32}{9} \frac{Z^2}{\pi^3 a_0^3} \frac{\chi 3^{/2}}{x^{3/2}}$.

Define particle exchange operator and find its eigen value.

OR

(1) Show $\langle \hat{A} \rangle_\Psi$ is constant of motion in Schrodinger picture.

(2) N-non-interacting bosons are in an infinite potential well defined by, **7**

$$\begin{aligned} V(x) &= 0 \qquad , 0 < x < \mathrm{a} \\ &= \infty \qquad , x < 0 \text{ and } x > \mathrm{a} \end{aligned}$$. Find the ground state energy of the system.

What would be the ground state energy if the particles are fermions ?

2. (A) Define annihilation and creation operations for simple harmonic oscillator with unit mass. Show that $[\hat{a}, \hat{a}^{+}] = 1$.

Define number operator. Find out eigen value of harmonic oscillator.

What is Fock state ? With necessary equations show that any excited state can be expressed in terms of ground state. **7**

OR

Write solution of this equation $\nabla^2 \vec{A}(\vec{r}, t) = \frac{1}{c^2}\frac{\partial^2 \vec{A}(\vec{r}, t)}{\partial t^2}$.

Using gauge transformation equation find out k^{th} component of electric field E_k and magnetic field B_k.

Show that average energy $\langle \varepsilon_k \rangle$ radiated from the cavity having volume V is given by energy of the harmonic oscillator with unit mass.

(B) Write eigen value equation for the coherent states. How one can calculate (Δp^2) and (Δq^2) in coherent states ?

Considering position $\hat{q}$ and momentum $\hat{p}$ are non-commutative. Show that the product of (Δq) with (Δp) is greater or equal to $\hbar^2/2$. **7**

OR

Write Hamiltonian for the atom placed in the external perturbative potential in terms of non-perturbative potential.

Using method separation of variables method, find out soultion of time dependent Schrodinger equation.

Using such time dependent solution, find out time dependent co-efficient of liner expansion in terms of matrix element of perturbative Hamiltonian.

3. (A) (1) If f(z) is single valued and analytic throughout a simply connected region R, and C is a close curve lying within region R and enclosing a single pole Z_0 then, $f(Z_0) = \frac{1}{2\pi i}\oint_c \frac{f(Z)}{Z - Z_0}\, dZ$. **7**

(2) Using above theorem show that n^{th} order derivative of an analytic function is given as $f^n(Z_0) = \frac{n!}{2\pi i}\oint_c \frac{f(Z)}{(Z - Z_0)^{n+1}}\, dZ$.

OR

Show that $\int_0^{2\pi} \frac{d\theta}{[a + b\cos\theta]^2} = \frac{2\pi}{(a^2 - b^2)^{3/2}}$; a > b **7**

(B) Using the method of contour integration, evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$. **7**

OR

(1) Evaluate $\oint_c \frac{(\sin Z)^6}{\left(Z - \frac{\pi}{6}\right)^3} dZ$.

(2) Find the residue of f(Z) $\frac{Z^4}{(Z-1)(Z-2)(Z-3)}$ at Z = 3. **7**

4. (A) Write a general form of an integral equation. Explain its classifications. Transform a second order differential equation into an integral equation. **7**

OR

Transform a given differential equation into an integral equation.

$Y''(x) + \omega^2 Y(x) = 0$ with Y(0) = 0 and Y(b) = 0.

(B) Describe separable Kernel method for solving an integral equation. **7**

OR

Describe Green's function for one dimensional problem. Show that a homogenous differential equation with non-homogeneous boundary conditions can be transferred as non-homogeneous equation with homogeneous boundary conditions. **7**

5. Answer the following questions : **14**

(1) $\hat{U}(t_1, t_2) = \hat{U}(t, t_1)\, \hat{U}(t_1, t_2)$. (True / False)

(2) What will be the value of half-lifetime if the value of Einstein co-efficient for induced absorption is 2×10^{-5} Sec^{-1} ?

(3) Find out <cosθ> over solid angle.

(4) Find out unknown states when lowering and raising operator operates on that unknown states and it gives $|2\rangle$ and $|4\rangle$ respectively.

(5) If $\vec{A} = r^2 + 2r + 1$, then find out $\vec{B}$.

(6) Show that $\left[\hat{a}^{+}, \hat{n}\right] = -\hat{a}^{+}$

(7) What do you mean by exchange degeneracy ?

(8) $G(x, \xi)$ is a continuous function of x then

$$\lim_{x \to \xi} G_1(x, \xi) = \lim_{x \to \xi} G_2(x, \xi). \text{ (True / False)}$$

(9) State the residue theorem.

(10) Write the Laurent Series Expansion.

(11) $f(Z) = \dfrac{1+Z}{1-Z}$. Represent this function in u + iv form.

(12) Evaluate $\displaystyle\oint_c \frac{(\sin Z)^6}{\left(Z - \frac{\pi}{6}\right)} dZ$.

(13) Show that the function $u = x^3 - 3xy^2$ is harmonic.

(14) Find the function $f(Z) = Z^2$ is analytic or not.
