Seat No. : _______________

# AB-120

**April-2016**

## B.Sc., Sem.-VI

## CC-307 : Statistics

## (Distribution Theory – II)

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) **All** questions carry equal marks.
(2) Scientific calculator is permitted.

1. (a) State Cauchy distribution. Let X has Cauchy $(0, \lambda)$, 0 is location parameter and $\lambda$ is scale parameter, obtain the distribution of 1/X. Obtain also characteristic function of the distribution of 1/X.

**OR**

State Laplace distribution. If $X_1$, $X_2$, $X_3$, $X_4$ are independent standard normal variates then show that $X_1 X_2 - X_3 X_4$ follows standard Laplace distribution.

(b) Let X a and Y are independent uniform U(0, l) variates. Show that Z = log(X/Y) follows standard Laplace distribution. Obtain mode of the distribution of Z.

**OR**

State lognormal distribution. Obtain coefficient of variation for lognormal distribution.

2. (a) State bivariate normal distribution of random variable (X, Y). Derive conditional distribution of X given Y = y.

**OR**

Obtain characteristic function of bivariate normal distribution. Hence deduce the marginal distribution of each variable.

(b) Let Z = (X, Y) ~N(1, –1, 4, 9, 0.2), find out (i) P(X > Y) (ii) distribution of 2X – 3Y (iii) distribution of X given Y = 2.

**OR**

The probability density function of bivariate normal distribution is given by $f(x, y) = C\exp\{-K(x^2 + y^2 - 2.6x + 2.6y - 0.6xy + 0.6)\}$

Determine (i) constants C and K (ii) Parameters of the distribution.

3. (a) State and prove Tchebychev's inequality. When the equality sign can be achieved ?

**OR**

State and prove Bernoulli law of large numbers.

(b) Sequence of independent random variables $X_k$, k = 1, 2, ...... have the distribution as

$P(X_k = \pm 2^k) = 2^{-(2k+1)}$ , $P(X_k = 0) = 1 - 2^{-2k}$

Verify whether the WLLN holds or not.

**OR**

Let X~N (4, 16). Using Tchebychev's inequality derive a lower bound for the $P(-1 < X < 9)$. Compare it with the actual value of the probability.

4. (a) State and prove Central Limit Theorem (CLT) for independent identically distributed random variables. Discuss its importance.

**OR**

State and prove Lindburg-Levy form of central limit theorem. How it differ from other forms of CLT.

(b) Let $X_1, X_2, \ldots .X_n$ are iid exponential variates with mean 2. Find the asymptotic distribution of $Y = \sum_{i=1}^{100} X_i$. Hence find $P(150 < Y < 260)$.

**OR**

Let $X_1, X_2, \ldots, X_n$ are Bernoulli variates with parameter 0.2. Let $Y = \sum_{i=1}^{100} X_i^2$.

Using CLT find $P(15 < Y < 25)$.

5. Answer the following :

(i) If X ~ Cauchy(2, 1.5), state the pdf of 3X + 2.

(ii) State additive property of Cauchy distribution.

(iii) State characteristic function of Laplace distribution.

(iv) State CDF of standard Laplace distribution.

(v) State mgf of log normal distribution.

(vi) If X ~ N (2.3) write the pdf of exp(X).

(vii) Write the pdf of standard bivariate normal distribution.

(viii) If X ~ N (2, 3) and Y ~ N(–2, 3), X and Y are independent, write the pdf of the random variable Z = (X,Y).

(ix) If $X \sim N_2(1, -1, 4, 9, 0.75)$, state the conditional variance of Y given X = 0.

(x) State convergence in probability.

(xi) State convergence in distribution.

(xii) Which types of convergence is used in WLLN ?

(xiii) State Liapounoff's theorem on CLT.

(xiv) State Cauchy-Shewhart's inequality.