Seat No. : _______________

AB-114

April-2016

B.Sc., Sem.- VI

CC-307 : Mathematics
Abstract Algebra – II

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry **14** marks.

(2) Figures to the right indicate marks of the question/sub-question.

(3) Notations are as usual.

1. (a) Define Ring and Division ring. In a ring (R, +, •) for a, b $\in$ R, prove that **7**

(i) a0 = 0a = 0

(ii) a(–b) = (–a)b = –(ab)

(iii) (–a) (–b) = ab, where 0 is the zero element of ring R.

OR

Define zero divisor in a ring. Show that a ring R is a ring without zero divisor if and only if cancellation laws hold good in R.

(b) Show that every field is an integral domain. Is converse true ? Justify your answer. **7**

OR

What is characteristic of a ring ?

Prove that if p is the characteristic of an integral domain D, then show that

$(a + b)^p = a^p + b^p$; for a, b $\in$ D

2. (a) Define subring. State and prove the necessary and sufficient condition for a non empty subset to be a subring of ring R. **7**

OR

Define an ideal in a ring. Prove that the only ideals of field F are {0} and F itself.

(b) If f is a homomorphism of a ring R onto a ring R' and I is kernel of homomorphism f, then prove that R' is isomorphic to quotient ring R/I. **7**

OR

Prove that every ideal is a subring in a ring. Is converse true ? Justify your answer.

3. (a) Find the G.C.D. of polynomials. **7**

$f(x) = x^4 + x^3 - 3x^2 - x + 2$ and $g(x) = x^4 + x^3 - x^2 + x - 2$ over the field of rationals. Express the G.C.D. as a linear combination of two polynomials.

OR

Prove that a polynomial domain F[x] over a field of F is a principal ideal ring.

(b) Obtain all rational roots of polynomial $2x^4 - 5x^3 - 2x^2 - 4x + 3$. **7**

OR

State Eisenstein criterion for irreducibility of polynomials.

Prove that the polynomial $x^{p-1} + x^{p-2} + x^{p-3} + \dots\dots\dots + x^3 + x^2 + x + 1$ is irreducible over the field of rational numbers where p is a prime.

4. (a) Prove that an ideal I of the ring of integers is maximal iff I is generated by some prime integer. **7**

OR

Prove that a ring R can be embedded in a ring R' with unity.

(b) An ideal I in a commutative ring R with unity is a prime ideal iff the quotient ring R/I is an integral domain. **7**

OR

Prove that the ideal $I = \langle x^3 - x - 1 \rangle$ is maximal ideal in $\mathbb{Z}_3[x]$.

5. Answer in short : (any **seven**) **14**

(i) Define the G.C.D. of two polynomials.

(ii) Is the ring $(\mathbb{Z}_6, +_6, \times_6)$ a field ?

(iii) Is the polynomial $8x^3 + 6x^2 + 9x + 24$ irreducible over $\mathbb{Q}$?

(iv) Give an example of a commutative ring with unity.

(v) What is the characteristic of the ring $(\mathbb{Z}_8, +_8, \times_8)$?

(vi) If I is an ideal in a ring R and $1 \in I$, then prove that I = R.

(vii) Define monic polynomial.

(viii) Is the ideal $\langle 6 \rangle$ prime ideal in the ring of integers ? Justify your answer.

(ix) Define unit ideal and null ideal.