Seat No. : _______________

NB-106

December-2015

B.Sc., Sem.-V

Core Course-301 : Statistics
(Distribution Theory-1)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : (1) **All** questions carry equal marks.

(2) Use of scientific calculator is allowed.

1. (a) State and prove Lack Memory Property of Geometric distribution.

OR

If X ~ NB (r,P) then find moment generating function, cumulant generating function and first three cumulants of X.

(b) Let the independent random variables X_1 and X_2 have the same geometric distribution. Obtain the conditional distribution of $X_1 \mid (X_1 + X_2 = n)$.

OR

The probability that a person can hit a target is 0.8. He gets a prize when he hits the target 4^{th} time. Find the probability that he will require more than 7 trials to get the prize.

2. (a) Consider a standard normal distribution truncated at both ends with cutoff points t_1 and t_2. Obtain p.d.f., mean, mode and variance of this distribution.

OR

Derive the p.d.f. of Geometric distribution truncated at X = a (a > 0). Also find its moment generating function, mean and variance.

(b) Derive and define Binomial distribution truncated at X = 0. Also find its characteristic function, mean and variance.

OR

Let $X \sim N(\mu,\sigma^2)$. Obtain the p.d.f. of truncated distribution from both the sides with lower cutoff point t_1 and upper cutoff point t_2. Also find mean and variance of this truncated distribution.

3. (a) Obtain the distribution of largest observation and smallest observation of order statistics.

OR

Obtain the distribution of range of order statistics.

(b) For the sample of n observations from the distribution with p.d.f.

$$f(x) = \frac{1}{b - a} ; a < x < b$$

obtain the distributions of largest observation and smallest observation.

Also obtain the probability density function of sample range R.

OR

Let $x_1, x_2, \ldots . x_n$ be a random sample from the distribution with p.d.f.

$$f(x) = e^{-x} ; 0 < x < \infty$$
$$= 0 \quad ; \text{otherwise}$$

Find the p.d.f. of (i) largest order statistic

(ii) smallest order statistic.

4. (a) Obtain Binomial distribution and Poisson distribution as a special case of Power series distribution.

OR

Obtain geometric and logarithmic series distribution as a special case of Power series distribution.

(b) For Power Series distribution, in usual notation, prove that

$$\mu_{r+1} = \theta \frac{d\mu_r}{d\theta} + r\mu_{r-1}\mu_2$$

OR

Define Power Series distribution and find mean and variance of power series distribution.

5. Answer the following objective questions.

(1) Write mean and variance of Geometric distribution with parameter p.

(2) Write m.g.f. and c.g.f. of geometric distribution.

(3) Write three conditions under which negative binomial distribution approaches Poisson distribution.

(4) Write p.m.f. and mean of Geometric distribution truncated at X = 0.

(5) Define truncated distribution at X = a and also write its probability function.

(6) Define : power series and order statistics.

(7) Write m.g.f. and characteristic function of power series distribution.
