Seat No. : _______________

NB-102

December-2015

B.Sc., Sem.-V

Core Course-301 : Mathematics

(Linear Algebra-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory and carry **14** marks.

(2) Right hand side figures indicate marks of the question/sub-question.

1. (a) If $T : U \to V$ is linear map, $v_0 \in R(T)$ and if $T(u) = \overline{0}_v$ has a nontrivial solution $u \neq \overline{0}_u$, then prove that the operator equation $T(u) = v_0$ has an infinite number of solutions. **7**

OR

If $T : U \to V$ and $S : V \to W$ are linear maps, then prove that the composition map $S \cdot T : U \to W$ also is a linear map.

(b) If a linear map $T : V_2 \to V_3$ is defined as $T(x_1, x_2) = (x_1, x_1 - x_2, x_1 + x_2)$, $\forall (x_1, x_2) \in V_2$, then solve the operator equation $T(x_1, x_2) = (5, 2, 8)$. **7**

OR

Find the dual basis of the basis $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ for the vector space V_3.

2. (a) Prove that a finite dimensional inner product space has an orthogonal basis. **7**

OR

State and prove the Cauchy-Schwarz inequality.

(b) If for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ the map $<, >$ is defined as $< x, y> = x_1 y_1 - x_1 y_2 - x_2 y_1 + 2 x_2 y_2$, then show that $<, >$ is an inner product on R^2. **7**

OR

Apply the Gram-Schmidt orthogonalization process to the basis $B = \{ (2, 1, 1), (1, 2, 1), (1, 1, 2)\}$ in order to get orthogonal basis for R^3.

3. (a) If $det : V^n \to R$ is a function satisfying the properties of the determinant, then prove the following : **7**

(i) $det(v_1, v_2, \ldots, v_i, \ldots, v_j, \ldots, v_n) = det(v_1, v_2, \ldots, v_i + 3v_j, \ldots, v_j, \ldots, v_n)$, for $i \neq j$.

(ii) $det(v_1, v_2, v_3, v_4, \ldots, v_n) = - det(v_1, v_3, v_2, v_4, \ldots, v_n)$.

OR

State and prove the Cramer's rule for solving a system of linear equations.

(b) If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 4 & 5 \\ 2 & 3 & 1 & 4 \\ 1 & 0 & -1 & 3 \end{pmatrix}$, then find *det*A by applying the Laplace Expansion about the second row of the matrix A. **7**

OR

If $A = \begin{pmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{pmatrix}$, then compute *det*A without expansion.

4. (a) Define eigen value and eigen vector of a linear operator $T : V \to V$. Also find eigen value and eigen vector of the linear map $T : V_2 \to V_2$ defined as $T(x_1, x_2) = (x_2, x_1)$, if exists. **7**

OR

State and prove the Cayley-Hamilton's Theorem.

(b) Verify the Cayley-Hamilton's theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}. **7**

OR

Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Also find the corresponding modal matrix.

5. Answer any **seven** of the following questions in short : **14**

(a) Define homogeneous and non-homogeneous operator equations.

(b) Define a linear functional and give one example of it.

(c) Define the space L(U, V) and an isomorphism.

(d) Define an inner product and give one example of it.

(e) Define orthonormal set and give one example of it.

(f) State the Laplace Expansion.

(g) Find *det*A if $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{bmatrix}$.

(h) Define a bilinear map and a quadric.

(i) Write an equation of a hyperboloid of two sheets.