Seat No. : _______________

# NM-101

**December-2015**

**B.Sc., Sem.-III**

**Core Course-201 : Statistics**

**(Random Variable and Probability Distribution Theory – I)**

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) All questions are compulsory and carry equal marks.

(2) Statistical tables and graph papers will be provided on request.

(3) Use of scientific calculator is allowed.

1. (a) Define the following terms :

(i) Random variable

(ii) Probability mass function

For a random variable X, p.m.f. is given as under :

$P(x) = (x+2) / 7, \quad x = 1, 2$

$= 0, \quad$ Otherwise

Find $P(X > 1)$, $P(0 < X < 3)$.

**OR**

(a) State and prove the properties of the distribution function of a random variable X. If a r.v X has a probability distribution as shown in the following table :

| **X** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| **P(*x*)** | 0 | 0.5 | 0.03 | 0.07 | 0.1 | 0.04 | 0.06 | 0.2 |

Determine $P(x < 5)$, $P(x \geq 4)$, $P(2 < x \leq 6)$

(b) Define probability density function.

If X follows a probability density function $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$

Then find (i) k (ii) distribution function (iii) $P(X < 0.4)$

**OR**

(b) For the continuous distribution of a random variable X with the probability density function $f(x) = \begin{cases} 2(1+x)/27, & 2 < X < 5 \\ 0, & \text{elsewhere} \end{cases}$

Find the distribution function and calculate F(2), F(4).

2. (a) Define Mathematical Expectation.

In usual notation prove that if X and Y are two independent random variables, then, E(XY) = E (X) E (Y).

**OR**

(a) Define terms : moments, moment generating function of a random variable X. State and prove properties of moment generating function.

(b) A random variable X has a probability density function

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

then, find E(X). Also, show that mode of this probability density function is same as E(X)/

**OR**

(b) In usual notations, prove $\mu_4 = k_4 + 3k_2^2$

3. (a) State probability mass function of binomial distribution.

A company makes electronic components for TV's. 5% fails the final inspection and need to be fixed. 120 components are inspected in one day. What is the expected number that fail in one day ?

**OR**

(a) For the Binomial Distribution, in usual notations, derive the recurrent relation for cumulants.

(b) If X ~ Po(m), in usual notations, prove that $k_{r+1} = \lambda \frac{dk_r}{d\lambda}$, r = 1, 2, 3,…

**OR**

(b) State and prove additive property of poisson distribution.

4. (a) Derive mean and variance of Rectangular Distribution.

**OR**

(a) Show that the mean of Beta distribution of $1^{st}$ kind is m/(m + n).

(b) Define the beta distribution of second kind. Obtain its mean and variance.

**OR**

(b) If a r.v. X has an Uniform Distribution U[0, 1], then obtain the pdf of –2log X.

5. Answer the following :

(a) State discrete and continuous types of random variables with one illustration of each.

(b) For a random variable X, p.m. f is given as under :

$P(x) = x - 1, \quad x =, 1, 2,$

$\quad = 0, \quad \text{otherwise}$

Find E(X).

(c) State the cumulant generating function of poisson distribution. Write its mean.

(d) What is the probability distribution of $Y = \sqrt{x}$, if a random variable X has Uniform distribution ?

(e) If the probability density function of a random variable X is $f(x) = a_{ae}{}^{-ax}$, $x > 0$, then identify the distribution and for what value of a, its become simple exponential distribution.

(f) State probability mass function of hyper geometric distribution and state one application of it.

(g) A random variable assumes three values –1, 0, 1 with probabilities 1/3, 1/2, 1/6 respectively, then find E(X) and E(2X + 4)