Seat No. : _______________

NS-124

December–2015

M.Sc., Sem.–I

403 : Statistics
(Estimation Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : All questions carry equal marks.

1\. (a) Define minimal sufficient statistic. State & prove Lehmann-Scheffe theorem on minimal sufficient statistic. Show that for Cauchy distribution with location parameter θ, no nontrivial minimal sufficient statistics exist.

OR

Define complete sufficient statistic. Discuss complete sufficient statistic in exponential family of distributions. Hence deduce complete sufficient statistic for (μ, σ^2) in case of normal distribution N(u., σ^2) based on a random sample of size n.

(b) Let $X_1, X_2, ..., X_n$ be a random sample from negative binomial distribution with parameters 1and p, $0 < p < 1$. Obtain minimal sufficient statistics for p. Check the completeness property of the statistic obtained by you.

OR

Let X_1 and X_2 be iid with discrete uniform distribution given by $P(X=x) = 1/N$, $x = 1, 2, ..., N$, $N \geq 1$. Obtain minimal sufficient statistic for N. Check whether the statistic obtained is complete.

2\. (a) State and prove Cramer-Rao inequality for the variance of unbiased estimator. Hence deduce it for MSE of biased estimator.

OR

State and prove Rao-Blackwell theorem. Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution with mean λ, $\lambda > 0$. Obtain UMVUE for $e^{-\lambda}(1 + \lambda)$.

(b) State and prove necessary and sufficient condition for unbiased estimator T for $g(\theta)$ to have a minimum variance at the value $\theta = \theta_0$. Hence deduce that the correlation coefficient between MVUE and any unbiased estimator is non-negative.

OR

Let $X_1, X_2, ..., X_n$ be a random sample from uniform distribution $U(\alpha, \beta)$, $\alpha < \beta$. Obtain UMVUE for α, β and $\beta - \alpha$.

3. (a) Prove or disprove: MLE is consistent. Check the property in case of normal distribution with mean μ, and variance 1 based on a random sample of size n.

OR

Let $X_1, X_2, ..., X_n$ be a random sample from the distribution given below. Obtain MLE of α. What do you conclude from your answer ?

$$f(x; \alpha, \beta) = \begin{cases} \frac{2x}{\alpha\theta}, & 0 \leq x \leq \theta \\ \frac{2(\alpha - x)}{\alpha(\alpha - \theta)}, & \theta \leq x \leq \alpha \end{cases}$$

(b) Show that MLE is asymptotically normally distributed statistic. Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with mean $\theta > 0$. Obtain mle of θ. Obtain its asymptotic distribution.

OR

Let $X_1, X_2, ..., X_n$ be a random sample from the exponential distribution with location parameter μ and scale parameter θ. Obtain MLE of the parameters. Hence obtain MLE of $\mu + 3\theta$, $\theta > 0$.

4. (a) What is $(1 - \alpha)100\%$ confidence interval estimation ? State its interpretation. How it differs from the point estimation ? Discuss with illustration.

OR

Discuss pivotal method of obtaining confidence interval with illustration.

(b) Let $X_1, X_2, ..., X_n$ be a random sample from the exponential distribution with mean $1/\theta$, $\theta > 0$. Construct 95% confidence interval for θ. Obtain expected length of the confidence interval.

OR

Discuss a general method of obtaining confidence interval with illustration.

5. Suggest correct single answer in the following :

(i) If T be any sufficient statistic for parameter θ then

$L(x, \theta) =$ _________.

(a) $g(t, \theta)h(x)$ (b) $g(\theta)h(t, x)$

(c) $g(t)h(x, \theta)$ (d) none of these

(ii) Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$, $\theta > 0$ distribution. And $T = X_{(n)}$. Then UMVUE of θ^2 is

(a) $\frac{3T^2}{n}$ (b) $\frac{n+2}{n}T^2$

(c) $\frac{n}{n+2}T^2$ (d) $\frac{n}{n+1}T^2$

(iii) Let $X_1, X_2, ..., X_n$ be a random sample from the pdf $f(x; \theta) = (\pi[1 + (x - \theta)^2])^{-1}$, $-\infty < x < \infty$. Define $T_1 = X_{(n)} - X_{(1)}$ and $T_2 = X_{(n)} + X_{(1)}$. Then

(a) T_1 is ancillary and T_2 is not a sufficient for θ

(b) T_2 is ancillary and T_1 is sufficient for θ

(c) T_1 is minimal sufficient and T_2 is complete statistic for θ

(d) T_1 and T_2 both are sufficient statistic for θ

(iv) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, \theta)$ normal distribution and $S = \sum_{i=1}^{n} (x_i - \bar{x})^2$. Then S is unbiased estimator of

(a) $n\theta$ (b) $n\theta^2$

(c) $(n-1)\theta$ (d) $(n-1)\theta^2$

(v) Let $X_1, X_2, ..., X_n$ be a random sample from the pdf $f(x; \theta) = \theta^2 x e^{-\theta x^2}$, $x > 0$. An unbiased estimator of θ is

(a) $\frac{X_1 + X_2 + X_3}{3}$ (b) $\frac{1}{3}\left(\frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3}\right)$

(c) $\left(\frac{X_1 + X_2 + X_3}{3}\right)^{-1}$ (d) $3\left(\frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3}\right)^{-1}$

(vi) If T is an unbiased and sufficient for θ then LogT for Logθ is

(a) unbiased and sufficient both

(b) unbiased only

(c) Neither unbiased nor sufficient

(d) sufficient only

(vii) Let $X_1, X_2, ..., X_n$ be a random sample from U(0, θ), θ > 0 uniform distribution Let $T = 2\bar{X}$ be an estimator of θ. Then mean square error of T is

(a) $\theta^2/12$ (b) $\theta^2/3$

(c) $\theta^2/12n$ (d) $\theta^2/3n$

(viii) Say True or False : MLE is always better than any other estimator.

(ix) Let $X_1, X_2, ..., X_n$ be a random sample from the U(0, θ), θ > 0 uniform distribution. State the UMVUE of logθ.

(x) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$ distribution, μ and σ^2 both are unknown. Which one is not a statistic ? Here $S^2 = \sum_{i=1}^{n} \left(x_{(i)} - \bar{x}\right)^2$.

(a) $\sum_{i=1}^{n} (x_i - \mu)^2$ (b) $\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2/n$

(c) $\sum_{i=1}^{n} (x_i - S)^2/n$ (d) $\bar{x}/S^2$

(xi) Define Fisher information contained in statistic.

(xii) Define ancillary statistic.

(xiii) State Basu's theorem.

(xiv) Give an example of non unique MLE.
