Seat No. : _______________

NQ-129

December-2015

M.Sc., Sem.-I

402 : Statistics

(Probability Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : All questions carry equal marks.

1. (a) Let $\{A_n\}$ be a monotonic sequence of events in σ-field G, then show that

$$P\left(\lim_{n \to \infty} A_n\right) = \lim_{n \to \infty} P(A_n)$$

OR

Define Probability space. Let $\{A_n\}$ be a sequence of events in a probability space (χ, G, P) then prove that

$P(\lim \text{Sup } A_n) \geq \lim \text{Sup } P(A_n)$

(b) Define Conditional Probability Measure. State and prove Baye's theorem.

OR

Show that Probability measure P is monotonic and subtractive.

2. (a) State and prove Jensen's Inequality.

OR

State Markov's Inequality. Derive Chebyshev's Inequality from Markov's Inequality.

(b) Define different kinds of convergence. Show that convergence almost surely implies convergence in Probability.

OR

Let $\{A_n\}$ be a sequence of events defined on probability space, (χ, G, P), then if

$\sum_{n=1}^{\infty} P(A_n) < \infty$, show that $P(\lim \text{Sup } A_n) = 0$.

3. (a) State and prove Chebyshev's weak law of large numbers for uncorrelated r.v.s.

OR

State and prove Kolmogorov's inequality.

(b) State and prove Inversion theorem on characteristic function.

OR

Discuss Kolmogorov's strong law of large numbers.

4. (a) State and prove Lindberg-Levy's form of central limit theorem.

OR

Describe Birth and Death process. Obtain the system of difference differential equation for a Birth and Death process.

(b) Discuss 'Pure Birth Process' in detail.

OR

Define 'Markov Chain'. Evaluate P(2) and P(3) for homogeneous Markov Chain with Transition Probability Matrix

$$P(1) = \begin{pmatrix} -2 & -5 & -3 \\ -5 & -1 & -4 \\ -1 & -2 & -7 \end{pmatrix}$$

Find also the probabilities of each state in every step transition. Assume the initial probabilities of state as 0.5, 0.3 and 0.2 respectively.

5. (a) Make correct choice from the following : (any **four**)

(i) Let {An} be a sequence of sets in χ, then $\lim_{n \to \infty}$ Int A_n = __________

(1) $\bigcap_{m=1}^{\infty} \bigcup_{n=m}^{\infty} A_n$ (2) $\bigcup_{m=1}^{\infty} \bigcap_{n=m}^{\infty} A_n$

(3) $\bigcap_{m=1}^{\infty} \bigcap_{n=m}^{\infty} A_n$ (4) $\bigcup_{m=1}^{\infty} \bigcup_{n=m}^{\infty} A_n$

(ii) Cauchy-Schwartz Inequality can be obtained from ___________.

(1) Markov's Inequality(2) Kolmogorov's Inequality

(3) Jensen's Inequality (4) Holder's Inequality

(iii) Let $\{A_i\}$ i = 1, 2…n be a sequence of events in a probability space. Then $P(\bigcup_{i=1}^{n} A_i) \leq$ ________.

(1) $\sum_{i=1}^{n} P(A_i) - 1$ (2) $\sum_{i=1}^{n} P(A_i)$

(3) $\sum_{i=1}^{n} P(A_i) + (n - 1)$ (4) $\sum_{i=1}^{n} P(A_i) - (n - 1)$

(iv) For n events A_i, i = 1, 2…n, __________ relations in probability must be satisfied for Mutual Independence.

(1) 2^n (2) n

(3) $2^n - 1$ (4) $2^n - n - 1$

(v) Let $\{A_n\}$ be a sequence of events in probability space such that

$P(A_n) = \frac{1}{2} P(A_{n-1})$, n = 2, 3…. $A_1 = \chi$, then P(lim Int A_n) = ________.

(1) 1 (2) $\frac{1}{2}$

(3) 0 (4) None of these

(b) Answer the following : (any **five**)

(1) Explain Pair-wise Independence and Mutual Independence.

(2) State Holder's Inequality.

(3) Show that characteristics function is uniformly continuous.

(4) State weak compactness theorem.

(5) State Kinchiru's weak law of large numbers.

(6) State Liapounov's form of central limit theorem.

(7) Give postulates of 'Poisson Process'.