Seat No. : _______________

# NQ-126

**December-2015**

**M.Sc., Sem.-I**

**402 : Physics**

**(Classical Mechanics – I & Statistical Mechanics)**

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) Symbols and terminology used have their usual meanings.
(2) Figures to the right indicate full marks.
(3) Assume suitable data wherever necessary.

1. (a) In case of tri-atomic molecule $CO_2$, find out three frequencies of oscillations and eigen vectors for them. **7**

**OR**

Explain normal coordinates. Find out kinetic energy and potential energy in terms of normal coordinates. Also obtain Lagrangian equation of motion in terms of normal coordinates.

(b) In case of coupled system made up of two simple pendulum joined by spring of force constant k, frequency of oscillations are given by $\omega_1 = \sqrt{\frac{g}{l}}$ and $\omega_2 = \sqrt{\frac{g}{l} + 2\frac{k}{m}}$. **7**

Find :

(1) Eigen vectors
(2) Generalised coordinates
(3) Normal coordinates

**OR**

In case of vibrating string of length L fixed at both the ends, obtain Lagrangian equation of motion. Explain how frequencies of different mode of oscillations can be found.

2. (a) Explain, how Lagrangian and Hamiltonian are not constant. For generating function $F(q_j, t)$, obtain relation between old Lagrangian L and new Lagrangian L'. For simple harmonic oscillator if generating function is given by $F = \frac{1}{2} mi\omega q^2$ then show that new Lagrangian is $L' = \frac{1}{2} [\dot{q} + i\omega q]^2$. **7**

**OR**

For the generating function $F_l$ ($q_j$, $Q_j$, t), obtain following relations :

(i) $p_j = \left(\frac{\partial F_l}{\partial q_j}\right)$

(ii) $p_j = -\left(\frac{\partial F_l}{\partial Q_j}\right)$

(iii) $K = H + \frac{\partial F_l}{\partial_t}$

Also, discuss motion of simple harmonic oscillator for the generating function $F_l = \frac{1}{2}$ m $\omega q^2$ [cot Q].

(b) For the generating function F($q_j$, $Q_j$), obtain condition for the transformation to be canonical. **7**

(1) Show that $Q = \log\left(\frac{1}{q}\sin p\right)$ and P = q cot p are canonical.

(2) Prove that $\frac{\partial}{\partial t}[U, V] = \left[\frac{\partial U}{\partial t} V\right] + \left[U \frac{\partial V}{\partial t}\right]$.

**OR**

(1) Using Poisson bracket, show that $q = \sqrt{2P}\sin Q$ and $p = \sqrt{2P}\cos Q$ are canonical.

(2) Show that $P = 2\left[1 + q^{1/2}\cos p\right]q^{1/2}\sin p$ and $Q = \log\left[1 + q^{1/2}\cos p\right]$ are canonical.

3. (a) Define Johnson noise. State and prove Nyquist theorem. **7**

**OR**

Define Brownian motion. Discuss the Langevin theory of Brownian motion.

(b) Derive Fokker-Planck equation. State its usefulness. **7**

**OR**

Derive Wiener-Khintchine theorem and show that for an exponentially decaying system, the rapid fluctuations rests faster than slowly varying fluctuations.

4. (a) Giving suitable example, define second order phase transition. Show that in the second order phase transition the second derivatives of the Gibbs function is discontinuous. **7**

**OR**

Show that in the first order phase transition the latent heat is non zero.

(b) Using two dimensional Ising models, show that in absence of externally applied magnetic field, the low temperature spontaneous magnetization of a ferromagnet is estimated as **7**

$$M_s = N\mu \tan h\left(\frac{\xi J M_s}{2N\mu k\, T}\right)$$

**OR**

(b) Explain :

(i) Critical indices

(ii) Order parameter.

5. Answer the following in brief : (Each carry **one** mark) **14**

(1) Find out unit of $\frac{\partial U}{\partial \theta_j}$.

(2) If natural frequency of oscillations of string is 0.2 $\pi$ radian/second, then find out frequency of only one normal mode of oscillation. If we have two normal modes of oscillations then what will be the relation between amplitudes $a_1$ and $a_2$ ?

(3) What is unstable equilibrium ?

(4) What will be the value of Poisson bracket $[q_j, q_k]$ ?

(5) What will be the unit of new Hamiltonian K in SI unit system ?

(6) If u = sin $x$ and v = cos $x$, then what will be the value of Poisson bracket $[u, v]_{x,y}$ ?

(7) What do you mean by canonical transformation ?

(8) Give the unit of diffusion coefficient.

(9) State the significance of auto correlation function

(10) Plot a graph of molar volume versus temperature in first order phase transition.

(11) Plot temperature dependence of order parameter in magnetic transition.

(12) What is the origin of shot noise ?

(13) Differentiate between zeroth order and first order Ising model.

(14) "One dimensional Ising chain can possess ferromagnetism"– True/False.

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