Seat No. : _______________

NO-113

December-2015

M.Sc., Sem.-I

401 : Statistics

(Matrix Algebra and Measure Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.
(2) Scientific calculator can be used.

1. (a) If A: $m \times n$ and B: $n \times m$ are two rectangular matrices with $m < n$, then in usual notations obtain $|AB|$. If $M = AA'$, then show that $|M| \geq 0$ if $m \leq n$.

OR

State and prove Laplace method of expansion for the determinant of a square matrix A of order n. Obtain determinant of the following matrix.

$$A = \begin{bmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{bmatrix}.$$

(b) Let $L: n \times n$ be a nonsingular matrix and $M: n \times r$ and $N: r \times n$ be two matrices such that $P = L + MN$ is a nonsingular matrix. Show that

$$P^{-1} = L^{-1} - L^{-1} M(I_r + NL^{-1} M)^{-1} NL^{-1}$$

Hence obtain inverse of the matrix $P = (P_{ij})$ where

$$P_{ij} = \lambda \text{ for } i \neq j,\ P_{ii} = r;\ i, j = 1, 2, \ldots, n$$

OR

If $P = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is non-singular matrix such that A is a non-singular matrix, then obtain $|P|$ and P^{-1}. Hence obtain determinant of a matrix $P = \begin{bmatrix} aI_m & cE_{mn} \\ dE_{nm} & bI_n \end{bmatrix}$.

2. (a) Define determinant of a matrix. Show that if in the elements of a row (or column) of a matrix $A = (a_{ij})$, a linear combination of the corresponding elements of remaining rows is added, the value of the determinant of resulting matrix remains unchanged.

OR

Define rank of a matrix. For any two matrices A: $m \times n$ and B: $n \times p$ show that

$$\rho(AB) = \rho(A^*AB) = \rho(ABB^*) = \rho(A^*ABB^*) = \rho(ABB^*A^*) = \rho(B^*A^*AB)$$

(b) Define system of homogeneous and non-homogeneous linear equations. Show that the system of linear equations $A\underline{x} = \underline{b}$ has a solution if and only if Rank (A) = Rank (A, $\underline{b}$). Derive the condition for $A\underline{x} = \underline{0}$ to have a non-trivial solution.

OR

Define reflexive g-inverse of a matrix A: m × n. In usual notations show that $B = Q\Delta^{-P}$ is a reflexive g-inverse of the matrix A.

3. (a) Define the following terms with appropriate example.

(i) Sample space

(ii) Class

(iii) Field

(iv) Sigma-field

Give an example of a class which is a field but not a sigma-field.

OR

Define sequence of sets. Show that given any sequence of sets we can construct monotonic sequences as well as sequences whose limits are limit infimum and limit supremum of the given sequence. Further, show that monotonic sequences are always convergent.

(b) Show that every union of arbitrary sets in a sigma-field can be expressed as a union of mutually disjoint sets in the same sigma-field.

OR

Define sequence of sets. Show that if $A_n \to A$, then $A_n^c \to A^c$.

4. (a) Define continuity of a set function. State and prove Continuity Theorem on measure μ.

OR

Define Measure. Write important properties of measure. Prove any two of them.

(b) Define Outer measure μ^*. Given a measure μ, how you will construct an Outer measure μ^*. Explain briefly with proper example.

OR

Define Outer measurable set. Give an example of outer measurable set. Show that a class of all outer measurable sets $\bar{G}$ is a field. Is $\bar{G}$ a sigma-field ?

5. (a) Choose the appropriate answer.

(1) A square matrix of order five has how many principal minors of order two ?

(A) 10 (B) 5

(C) 6 (D) 25

(2) If A : n × n is an idempotent matrix of rank 'r', then its characteristic roots are

(A) r times '0' and (n-r) times '1'

(B) r times '1' and (n-r) times '0'

(C) r roots are positive and (n-r) roots are negative

(D) None of the above

(3) Let X : m × n be a matrix with m ≤ n. If all the m rows of X are linearly Independent, then the number of linearly dependent columns of X is

(A) n (B) m

(C) n-m (D) m–n

(4) If A is a skew-symmetric matrix of order three, then which of the following Statement is not true ?

(A) A is a singular matrix

(B) One characteristic root of A is zero

(C) Two characteristic roots are imaginary

(D) A is a non-singular matrix

(5) For the following system of equations, which of the statement is not true ?

$2x_1, + 3x_2 = 8$

$4x_1, + 6x_2 = 16$

(A) This system of two equations in two unknowns has infinitely many solutions.

(B) Specifically $x_1 = \alpha$, $x_2 = (8 - 2\alpha)/3$ for any $\alpha \in R$ will be a solution.

(C) This system of two equations in two unknowns has no solutions.

(D) This system of two equations in two unknowns is consistent.

(6) If $A = \begin{bmatrix} 1 & 3 \\ 5 & 10 \end{bmatrix}$, then $|A^{-1}|$ is

(A) – 5 (B) 5

(C) – 1/5 (D) None of these

(7) If T n × n is a lower triangular matrix with positive diagonal elements, then determinant of matrix TT' is given by

(A) $\prod_{i=1}^{n} t_{ii}$ (B) $\prod_{i=1}^{p} t_{ii}^2$

(C) $\prod_{i=1}^{p} t_i^2$ (D) $\prod_{i=1}^{n} t_i^2$

(8) Let A be a square matrix of order n. The characteristic equation of the matrix A is

(A) $|A + \lambda I_n| = \sum_{j=0}^{n} c_j \lambda^{n-j}$

(B) $|A - \lambda I_n| = \sum_{j=0}^{n} c_j (-\lambda)^{n-j}$

(C) $|A - \lambda I_n| = \sum_{j=0}^{n} (-c_j) \lambda^{n-j}$

(D) $|A + \lambda I_n| = \sum_{j=0}^{n} c_{n-j} \lambda^{j}$

(9) Let A : $m \times n$ be a rectangular matrix. The rows of A are linearly dependent if and only if

(A) $A\underline{d} = \underline{0}$, $\underline{d} \neq \underline{0}$

(B) $\underline{d}' A = \underline{0}$, $\underline{d} \neq \underline{0}$

(C) $AD = 0$, $D \neq 0$

(D) None of these

(10) For matrices A and B, AB = 0 implies

(A) A = 0 or B = 0

(B) A = B = 0

(C) $A \neq B \neq 0$

(D) All of these three

(b) Answer the following questions.

(1) Give an example of a Skew-Hermitian matrix of order three. Write its determinant.

(2) Give an example of an Idempotent matrix of order three. Write rank, trace and determinant of the matrix you had written. (Except identity matrix).

(3) Give an example of a class which is a field as well as ring.

(4) Write an example of monotonically decreasing sequence.