Seat No. : _______________

NH2-109

December-2015

M.Sc., Sem.-III

505 : Mathematics

(Functions of Several Variables – II)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Let $g(s, t) = (t^2 - s^2)e_1 + (s^2 + t^2)e_2$, $s > 0, t > 0$. **7**

Let $A = \{(x, y) : 4 < x + y < 8, y - x > 0, x > 0\}$. Show that g is regular and evaluate

$$\int_A y^{-1}\, dV_2(x, y)$$

OR

(a) Let $A = \{(x, y) : x^2 + y^2 \leq 4, x \geq 0\}$.

Evaluate $\int_A y^2\, dV_2(x, y)$ by introducing polar coordinates.

(b) Answer any **two** : **4**

(i) Evaluate $\int_0^1 dx \int_0^1 \exp(x - y)\, dy$

(ii) Let $f(x, y) = x \exp(x^2 + y^2)$. Is the point (0, 3) in the support of f ?

(iii) Find the area of the triangle with vertices e_1, $e_1 + e_2$, $3e_1 + 4e_2$.

(c) Answer **all**. **3**

(i) Let $f(x) = \exp(x)$. Let A = [3, 8]. Is f integrable over A ?

(ii) Define a null set.

(iii) Let $A = [0, 1] \times [1, 3] \times [3, 6] \times [6, 10]$.

Find $V_4(A)$.

2. (a) Let $n = 4$, **7**

Let $h_1 = e_1 + e_2$, $h_2 = e_1 + e_2 + e_3$, $h_3 = e_1 + e_2 + e_4$. Show that the vectors h_1, h_2, h_3 are linearly independent in E^4.

Let $k_1 = 2e_1 + 2e_2 + e_3$, $k_2 = 2e_1 + 2e_2 + e_4$, $k_3 = 2e_1 + 2e_2 + e_3 + e_4$. Show that the vectors k_1, k_2, k_3 span the same vector subspace of E^4 as the vectors h_1, h_2, h_3.

OR

(a) Write down (without proof) the standard basis for the vector space E_r^4, $r = 1, 2, 3, 4$.

Let $h_1 = e_2 + e_3$, $h_2 = e_2 + e_3 + e_4$, $h_3 = e_1 + e_2 + e_3$. Show that the vectors h_1, h_2, h_3 are linearly independent in E^4.

(b) Answer any **two** : **4**

(i) Let S be the 3-simplex in E^4 with vertices $e_1, e_3, e_4, e_1 + e_3 + e_4$. Find $V_3(S)$.

(ii) Find the area of the triangle with vertices $2e_1$, $2e_1 + e_2 - e_3$, $3e_1 + e_2$.

(iii) Give two different frames for E^3.

(c) Answer **all** : **3**

(i) Simplify $e_3 \wedge e_5 \wedge e_{24}$.

(ii) Evaluate the scalar product $(e^1 + e^2) \cdot (e_1 + e_2)$

(iii) Simplify $(2e^2 - e^3) \wedge (3e^1 + e^3)$.

3. (a) Let $n = m = 3$, **7**

$g(s, t, u) = (s \cos(t))e_1 + (s \sin(t))e_2 + ue_3$. Find $(y\, dy \wedge dz)^{\#}$.

OR

(a) Show that if w and ξ are closed differential forms, then $w \wedge \xi$ is closed. Show that if w is closed and ξ is exact, then $w \wedge \xi$ is exact.

(b) Answer any **two** : **4**

(i) Find the exterior differential of $\sin(xy^2)\, dx \wedge dz$

(ii) Let $n = 3$, find $e_i \times e_j$ for all pairs $i, j = 1, 2, 3$.

(iii) Find a 2-form ξ such that

$$d\xi = dx \wedge dy \wedge dz$$

(c) Answer **all** : **3**

(i) Let n = 4. Find $*(e_{123} + e_{124} + e_{134} + e_{234})$

(ii) Let n = 3. Define the curl of a 1-form w.

(iii) Find a non-zero vector in E^2 orthogonal to $e_1 + 2e_2$.

4. (a) Let $A = \{(x, y, z) : y = x^2 + z^2, y \le 4\}$ oriented so that $O^{13}(x, y, z) < 0$ **7**

Evaluate $\int_{A^\circ} z\, dx \wedge dy$

OR

(a) Find the area of $A = \{(x, y, xy) : x^2 + y^2 \le 1\}$ **7**

(b) Answer any **two** : **4**

(i) Let $g : E^3 \to E^4$ be given by $g(s, t, u) = se_1 + te_2 + ue_3 + tue_4$.

Find $J_g(s, t, u)$

(ii) Define an orientable manifold

(iii) Let $S = \{(x, y, z) : 2x + 3y + 4z - 5 = 0\}$. Give a coordinate system for S.

(c) Answer **all** : **3**

(i) Give a map $g : E^1 \to E^3$ of class $C^{(1)}$. (Do not prove)

(ii) Give a map $g : E^1 \to E^3$ which is univalent (Do not prove)

(iii) Give a map $g : E^1 \to E^3$ for which $J_g(s) > 0$ for every $s \in E^1$.

5. (a) Evaluate $\int_{\partial\Sigma^+} z^2\, dx \wedge dy$, where Σ is the standard 3-simplex. **7**

OR

Let n = 3 and $D = \{(x, y, z) : x^2 + y^2 < z^2, 0 < z < 1\}$.

Evaluate $\int_{\partial D^+} (x + z)\, dx \wedge dy$.

(b) Answer any **two** : **4**

(i) Let n = 2 and assume that D is a regular domain. Show that $V_2(D) = -\int_{\partial D^+} y\, dx$.

(ii) Give an example of a regular domain in E^2. (Do not prove).

(iii) State (without proof) Stoke's formula.

(c) Answer **all** : **3**

(i) Let $D = \{(x, y) : 4 < x^2 + y^2 < 9\}$. Find the boundary of D. (Do not prove).

(ii) Let $D = \{(x, y, z) : x^2 + y^2 - z^2 < 1\}$. Is D bounded ?

(iii) Let $M = \{(x, y, z) : x^2 + y^2 + z^2 = 2\}$. Give a unit vector normal to M at (1, 1, 0)