Seat No. : _______________

NB-135

December-2015

M.Sc. Sem. – III

501 : Statistics

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Attempt **all** questions.

(2) **All** questions carry equal marks.

1. (a) State and prove necessary and sufficient part of NP lemma for randomized test.

OR

Exemplify the statement: The distribution specified under the null and alternative hypothesis not necessarily belong to the same family of the distributions for the application of NP lemma.

(b) Consider a population with three kinds of individuals labelled 1, 2 and 3. Suppose the proportion of individual of the three types are given by $p(k, \theta)$; $k = 1,2,3$; where $0 < \theta < 1$ and

$$p(k, \theta) = \begin{cases} \theta^2, & \text{if } k = 1 \\ 2\theta(1-\theta), & \text{if } k = 2 \\ (1-\theta)^2, & \text{if } k = 3 \end{cases}$$

Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from this population. Find the MP test for testing $H: \theta = \theta_0$ versus $K : \theta = \theta_1$, $0 < \theta_0 < \theta_1 < 1$.

OR

Let $X_1, X_2, \ldots\ldots, X_n$ be i.i.d. $N(\mu, 1)$ variates and independently $Y_1, Y_2, \ldots\ldots Y_n$ be i.i.d. $N(\mu, 4)$ random variables. Derive UMP test of size α to test $H: \mu = 0$ versus $K: \mu > 0$ based on the combined sample. Find also the power function of the test.

2. (a) State and prove theorem on UMP test.

OR

Define boundary set, α-similar test. Let Φ be any unbiased test of level α for testing $H : \theta \in \Omega_H$ versus $K : \theta \in \Phi\ \Omega_k$. Suppose that the function $E(\Phi(x))$, $\theta \in \Omega$ is continuous in θ then prove that Φ is α-similar on boundary set Λ.

(b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from the uniform distribution $U(0, \theta)$, $\theta > 0$. Derive UMP test of size α to test $H : \theta = \theta_0$ versus $K: \theta \neq \theta_0$. Hence find $(1-\alpha)$ 100% UMA confidence interval for θ.

OR

Define test with Neyman structure. State and prove necessary and sufficient condition for all α- similar tests to have Neyman structure with respect to T, T be sufficient statistic involved in the exponential family of distributions.

3. (a) Describe SPRT. Derive stopping bounds of SPRT. Show that $0 < B < 1 < A < \infty$

OR

Derive ASN and OC functions of SPRT.

(b) Consider the SPRT procedure as follows :

Continue the process if, $-\left(\frac{n+1}{2}\right) < \sum_{i=1}^{n} X_i < \left(\frac{n+2}{2}\right)$, where X_i's being the successive observations for testing H : P(X = – 1) = P(X = 1) = P(X = 2) = 1/3 versus K: P(X = –1) = P(X = 1) = 1/4; P(X = 2) = 1/2. What is the probability that the procedure will terminate under K on or before second stage ?

OR

For a given sequence of observations from Poisson distribution with mean $\lambda > 0$, derive SPRT to test H : $\lambda = \lambda_0$ versus K: $\lambda = \lambda_1$, $\lambda_1 < \lambda_0$. Also obtain OC and ASN function for the test.

4. (a) What is LRT ? For large sample tests show that under LRT the distribution of $-2 \log\lambda(X)$ is chi square with k d.f. for testing H: $\theta = \theta_0$ versus K : $\theta \neq \theta_0$, θ_0 is specified. What should be the value of k ?

OR

Derive LRT for testing H : $\sigma = \sigma_0$ versus K: $\sigma = \sigma_1$ in case of $N(\mu, \sigma^2)$ distribution, μ is known, based on a random sample of size n.

(b) Describe fully Kolmogorov Smirnov test for goodness of fit.

OR

Describe suitable non-parametric test for testing the equality of means of k independent groups.

5. Answer the following :

(i) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $N(0, \sigma^2)$ distribution. Consider the MP test to test $H : \sigma = \sigma_0$ versus $K : \sigma = \sigma_1$ $(\sigma_1 > \sigma_0)$.

(1) The BCR is given by $\sum_{i=1}^{n} X_i^2 \geq k;\ k \in R^+$

(2) Under H, $\sum_{i=1}^{n} X_i^2 / \sigma_0^2 \sim \chi^2_{(n-1)}$ can be used to find constant k in 1.

Which of the statements given above is/are true ?

(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

(ii) Define test function.

(iii) Define level of significance.

(iv) Define UMP test

(v) Define unbiased test.

(vi) Let $X \sim N(0, \sigma^2)$, and Y has exponential distribution with mean $2\sigma^2$ and X and Y are independent. We want to test $H : \sigma^2 \leq 1$ versus $K: \sigma^2 > 1$ at level α. Which of the following is true ?

(A) UMP test does not exist

(B) UMP test reject H; when $X^2 + Y$ is large

(C) UMP test is chi square test

(D) UMP test reject H when $X^2 + Y$ is small

(vii) To test the equality of two variances the appropriate non parametric test is

(A) Chi square test (B) F-test

(C) The Kruskal-Wallis test (D) The Siegel-Tukey test

(viii) State the approximate distribution of LRT test statistic to test $H : \theta_1 = \theta_2 = \ldots. = \theta k$ versus $K: \theta_1 \neq \theta_2 \neq \ldots. \neq \theta\, k$.

(ix) State MLE property.

(x) Let $\phi(x) = \begin{cases} 1, & \text{if } X_1 \geq 2 \\ 0, & \text{e.w.} \end{cases}$ be the test function to test H : $\lambda \leq 1$ versus K : $\lambda >$ 1 for Poisson distribution with mean λ. Find size of the test.

(xi) The K-S test procedures are based on

(A) vertical deviations between the observed and expected cumulative distribution functions.

(B) horizontal deviations between the observed and expected cumulative distribution functions.

(C) both horizontal and vertical deviations between the observed and expected cumulative distribution functions.

(D) None of the above

(xii) Staples manufacturing company claims that their latest machines put 1000 staples in a box on an average. We doubt its claim and wish to test the company claim. It is known that the population standard deviation is given by 7. Let a sample of 81 boxes gives average of staples = 997.

(A) Set null and alternative hypotheses.

(B) State test function

(C) Calculate the value of test statistic.