Seat No. : _______________

ZC-117

April-2014

M.Sc. Sem. IV

MAT-508 : Mathematics (Fourier Analysis)

Time : 3 Hours] **[Max. Marks : 70**

1. (A) Attempt any **one** : **7**

(1) State and prove the uniqueness theorem for continuous functions.

(2) Show that C is not dense in L^∞.

(B) Attempt any **two** : **4**

(1) Does there exist a non-constant function $f \in L^1$ such that

$\hat{f}(m + n) = \hat{f}(m) + \hat{f}(n)$ for all integers m and n ?

(2) If $f \in L^1$ then show that

$$\widehat{T_a f}(n) = e^{-ina} \hat{f}(n).$$

(3) If $f \in L^\infty$, then show that $|f(x)| \le \|f\|_\infty$, for almost every x.

(C) Answer in brief : **3**

(1) If $f(x) = 2 \cos x + 2$, then what is $\hat{f}(2)$?

(2) Prove or disprove : The Fourier transform map $T : L' \to l_\infty(\mathbb{Z})$ is onto.

(3) Give an example of a discontinuous function f such that $\hat{f}(1) = 1$.

2. (A) Attempt any **one** : **7**

(1) Let $\{K_n\}$ be an approximate identity. Then show that

$$\lim_{n \to \infty} \|K_n * f - f\|_\infty = 0, \forall f \in C.$$

(2) If γ is a non-trivial complex continuous algebra homomorphism between L1 and $\mathbb{C}$, then show that its Kernel is a maximal ideal in L^1.

(B) Attempt any **two** : **4**

(1) Let $1 \le p \le \infty$ and q be the conjugate index of p. If $f \in L^p$ and $g \in L^q$ then show that $f * g$ is continuous.

(2) If $f \in L^1$ and g is absolutely continuous then show that $f * g$ is absolutely continuous.

(3) Show that L^1 does not have identity with respect to convolution.

(C) Answer in brief : **3**

(1) Prove or disprove : If $f * f * f = f$, then f is a trigonometric polynomial.

(2) Show that convolution is associative.

(3) Give an example of an approximate identity in L^1.

3. (A) Attempt any **one** : **7**

(1) State and prove Weierstrass theorem for continuous functions using the Fejer's theorem.

(2) If $f \in L^1$, then prove that

$$\int_a^b f(x)dx = \hat{f}(0)(b-a) + \sum_{n \neq 0} \hat{f}(n) \frac{e^{inb} - e^{ina}}{in}.$$

(B) Attempt any **two** : **4**

(1) If $g \in L^\infty$ and $\hat{g}(n) = O(1/n)$, then show that $\{\|S_N g\|_\infty\}$ is a bounded sequence.

(2) Given $\delta > 0$, show that there exists $M > 0$, such that $|D_N(x)| \le M$, for all N and $\delta \le |x| \le \pi$.

(3) If for a trigonometric series $\sum c_n e^{inx}$, its cesaro means converge in L^1 norm to f, then show that $\sum c_n e^{inx}$ is a Fourier series of f.

(C) Answer in brief : **3**

(1) State Fejer's theorem.

(2) State any one conditions under which Cesaro summability implies summability.

(3) Show that $\int_{-\pi}^{\pi} F_N(x)dx$ is constant for all N.

4. (A) Attempt any **one** : **7**

(1) If $a_n \downarrow 0$ and (a_n) is convex then prove that $\sum a_n \cos nx$ is a Fourier series of $f(x) = \sum_{n=0}^{\infty} (n + 1) \Delta^2 a_n \frac{1}{2} F_n(x)$.

(2) If (a_n) is quasi-convex and bounded then show that the sequence $(n\Delta a_n)$ is bounded. Also show that if (a_n) is quasi-convex and convergent then the sequence $(n\Delta a_n)$ is convergent.

(B) Attempt any **two** : **4**

(1) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n} \sin nx$.

(2) Let $\delta > 0$. If $a_n \to 0$ and $\sum|\Delta a_n| < \infty$, then show that the cosine series $\sum a_n \cos nx$ converges uniformly in $\delta \leq |x| \leq \pi$.

(3) If $a_n \downarrow 0$ and $\sum \frac{a_n}{n} = \infty$, then prove that $\sum a_n \sin nx$ is not a Fourier series.

(C) Answer in brief : **3**

(1) Is $a_n = \frac{n}{n + 1}$ convex ?

(2) Show that the Fourier transform map $T : L^1 \to C_0(\mathbb{Z})$ is not onto using the open mapping theorem.

(3) True or False : If $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$, then f is continuous.

5. (A) Attempt any **one** : **7**

(1) Prove or disprove : $L^3 \subseteq L^1 * L^3$.

(2) State the Uniform Boundedness theorem and using it show that there exists a function which is continuous at 0 but whose Fourier series diverges at 0.

(B) Attempt any **two** : **4**

(1) If f is of bounded variation then show that $\{n\hat{f}(n)\}$ is a bounded sequence.

(2) State (only) some of the consequences of Jordan's theorem.

(3) Let $f \in L^1$ and s be any complex number. If for some positive δ,

$$\int_0^{\delta} \frac{f_s^*(y)}{y}\, dy < \infty, \text{ then show that } S_N f(x) \to s.$$

(C) Answer in brief : **3**

(1) State Jordan's theorem.

(2) True or False : $L^1 * C = C$.

(3) True or False : $L^2 * L^2 = L^2$.