Seat No. : _______________

LD-106

April-2014

B.Sc., Sem.-VI

CC-307 : Statistics

(Distribution Theory-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.

(2) Use of scientific calculator is allowed.

1. (a) Obtain characteristic function of standard Cauchy distribution. Also find the distribution of the arithmetic mean $\bar{X}$ of sample $X_1, X_2, \ldots, X_n$ of independent observations from a standard Cauchy distribution.

OR

Obtain distribution function of log-normal distribution and hence find probability density function of a log-normal variate. Obtain the expression for r^{th} moment about origin and find mean and variance for log-normal distribution.

(b) If X and Y are i.i.d. N(0, 1) then find the distribution of $\frac{X}{Y}$ and identify it.

OR

Find r^{th} moment about origin for laplace distribution with two parameters and also find mean and variance for it.

2. (a) If $(X, Y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then obtain the expression for its moment generations function and hence find $E(X^2)$, $E(Y^2)$ and $E(XY)$.

OR

If $(X, Y) \sim BN(5, 10, 1, 25, \rho)$ and if $P(4 < Y < 16 \mid X = 5) = 0.954$ then find the value of ρ where $\rho > 0$ and $\frac{1}{\sqrt{2\pi}} \int_0^2 e^{-\frac{1}{2}x^2}\, dx = 0.477$

(b) If the joint p.d.f. of X and Y is $f(x, y) = k \cdot \exp\left[-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right]$; $-\infty < x, y < \infty$ then find the value of k and the distribution of $Q = \frac{x^2 - 2\rho xy + y^2}{1 - \rho^2}$.

OR

If two dimensional random variable (X, Y) has standard bivariate normal distribution then show that (X + Y) and (X – Y) are independently distributed and also write the distribution of (X + Y) and (X – Y)

3. (a) State and prove Chehychev's inequality and generalized form of Chehychev's inequality.

OR

State and prove weak law of large numbers and Bernoulli's weak law of large numbers.

(b) An unbiased coin is tossed 100 times. Show that the probability that the number of heads will be between 30 and 70 is greater than 0.93.

OR

(i) Let $\{X_K\}$ be mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = x_1 + x_2, + \ldots\ldots + x_n$, prove that the law of large numbers does not hold for the sequence $\{S_n\}$.

(ii) Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$ where the random variables X_n are independent and X_n takes the values 1 and 0 with corresponding probabilities P_n & $(1 - p_n)$ $\forall$ n = 1, 2, 3…..

4. (a) State and prove Lindberg-Levy form of central limit theorem.

OR

If $X \sim P(\lambda)$ then show that $\frac{X - \lambda}{\sqrt{\lambda}} \sim N(0, 1)$ for large λ.

(b) Let Y denote the sum of item of a random sample of size 12 from a distribution having p.m.f. $p(x) = \frac{1}{6}$; $x = 1, 2, \ldots. 6$

$= 0$; Otherwise

Compute an approximate value of $\Pr[36 \leq Y \leq 48]$ considering that sample size n = 12 is large where $\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{1}{2}x^2} dx = 0.3413$

OR

Let $\bar{X}$ denote the mean of a random sample of size 75 from the distribution whose p.d.f. is given by

$$f(x) = 1 \quad ; \quad 0 \leq x \leq 1$$
$$= 0 \quad ; \quad \text{Otherwise}$$

Compute $P_r[0.45 < \bar{X} < 0.55]$ where $\frac{1}{\sqrt{2\pi}} \int_0^{1.5} e^{-\frac{1}{2}x^2} dx = 0.4332$

5. Answer the following questions :

(1) Write statement of Liapounoff's central limit theorem.

(2) Define convergence in probability.

(3) If $(X, Y) \sim B\,N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then what is the value of $E(X / Y = y)$ and $V(X \mid Y = y)$?

(4) Write the expression of c.g.f. for $B\,N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ distribution and find $k_{1,1}$.

(5) Write the mean deviation about mean and variance of standard laplace distribution.

(6) If X is a standard Cauchy variate then what is the distribution of X^2 ? Also write the p.d.f. of X^2.

(7) Define standard Laplace distribution and write its characteristic function.