Seat No. : _______________

XZ-112

April-2013

M.Sc. Sem.-IV

508 – MATHEMATICS

(Fourier Analysis)

Time : 3 Hours] **[Max. Marks : 70**

1. (A) Attempt any **one** : **7**

    (1) Prove that the set off all trigonometric polynomials is dense in C and in L^p for $1 \le p < \infty$.

    (2) Show that C is not dense in L^∞.

   (B) Attempt any **two** : **4**

    (1) If $1 \le p < q < \infty$, and $f \in L^q$, then show that $\| f \|_p \le \| f \|_q$.

    (2) If f is absolutely continuous then show that $\widehat{Df}(n) = in \hat{f}(n)$.

    (3) If $f \in L^1$ then show that

$$\hat{\bar{f}}(n) = \overline{\hat{f}(-n)} .$$

   (C) Answer in brief : **3**

    (1) State any one consequence of the uniqueness theorem.

    (2) State the Riemann-Lebesgue lemma.

    (3) Define convolution in L^1.

2. (A) Attempt any **one** : **7**

    (1) Let $f \in L^1$. Show that

     (i) If $g \in C^1$,then $f * g \in C^1$;

     (ii) If g is absolutely continuous then $f * g$ is absolutely continuous.

    (2) If γ is a non-trivial complex continuous algebra homomorphisms between L^1 and $\mathbb{C}$, then show that there exists a unique positive integer N such that $\gamma(f) = \hat{f}(N)$, for every $f \in L^1$.

(B) Attempt any **two** : **4**

(1) If $f \in L^1$ and g is of bounded variation then show that $f * g$ is of bounded variation.

(2) Show that L^1 does not have identity with respect to convolution.

(3) True or False : If for f, $g \in L^1$, $f * g \equiv 0$, then at least one of the functions f and g is a trigonometric polynomial.

(C) Answer in brief : **3**

(1) Show that convolution is commutative.

(2) Give an example of an idempotent element in L^1.

(3) Show that if $f * f = f$, then f is a trigonometric polynomial.

3. (A) Attempt any **one** : **7**

(1) State and prove localisation principle.

(2) State and prove Fejer's theorem.

(B) Attempt any **two** : **4**

(1) If $\sum x_n$ is summable to 0 then show that the series is cesaro summable to 0.

(2) Show that the uniqueness theorem follows from Fejer's theorem.

(3) If for a trigonometric series $\sum c_n e^{inx}$, its cesaro means converge in L^1 norm to f, then show that $\sum c_n e^{inx}$ is a Fourier series of f.

(C) Answer in brief. **3**

(1) State any one condition under which Cesaro summability implies summability.

(2) State any one consequence of localization principle.

(3) Show that $\sigma_N f = f * F_N$.

4. (A) Attempt any **one** : **7**

(1) If $a_n \downarrow 0$ and $na_n = 0(1)$, then show that $\sum a_n \sin nx$ converges boundedly in $[-\pi, \pi]$.

(2) If (a_n) is convex and bounded, then prove that (a_n) is decreasing and $n\Delta a_n \to 0$. Further, show that (a_n) is quasi-convex.

(B) Attempt any **two** : **4**

(1) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1} \sin nx$.

(2) If $a_n \to 0$ and $|\Delta a_n| < \infty$, then show that the sine series $\sum a_n \sin nx$ converges everywhere in $[-\pi, \pi]$.

(3) Prove or disprove : Every decreasing and bounded sequence is of bounded variation.

(C) Answer in brief : **3**

(1) Is $a_n = \frac{n}{n+1}$ convex ?

(2) True or False : If $a_n \downarrow 0$, then $\sum a_n \cos nx$ converges everywhere.

(3) True or False : If $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n \log (n+1)}$, then f is continuous.

5. (A) Attempt any **one** : **7**

(1) Prove that $C \subset L^1 * C$. Is it true that $L^1 * C \subset C$? Give reason for your answer.

(2) State the Uniform Boundedness theorem and using it show that there exists a function which is continuous at 0 but whose Fourier series diverges at 0.

(B) Attempt any **two** : **4**

(1) If f is of bounded variation then show that $\{n \hat{f}(n)\}$ is a bounded sequence.

(2) If (b_n) is a sequence of non-negative real numbers converging to 0, then show that there exists a sequence (a_n) of non-negative real numbers such that :

(i) $\sum a_n = \infty$ (ii) $\sum a_n b_n < \infty$ (iii) $\sum \frac{a_n}{n} < \infty$

(3) If $f \in L^1$ then show that $\sum_{n \neq 0} \frac{\hat{f}(n) e^{inx}}{n}$ converges uniformly.

(C) Answer in brief : **3**

(1) Give a necessary and sufficient condition under which a function $f \in L^2$ can be factorised as $g * h$ with $g, h \in L^2$.

(2) State Jordan's theorem.

(3) True or False : $L^1 * L^\infty = C$.