XX-137

M. Sc. (Sem. IV)
April-2013

507 – Statistics

(Time Series Analysis and Applications)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Define time series. Discuss variate difference method and show how (i) the trend component of a time series can be estimated and (ii) the variance of the random component is estimated using this method. State clearly the assumptions underlying this method and explain with illustration when the method fails. **7**

OR

Define stationary time series. How do you test the stationarity of the given time series ?

(b) Distinguish between

(i) Ordinary and stationary time series

(ii) RWM with drift and without drift

(iii) Trend stationary and difference stationary stochastic process. Why random walk is said to have an infinite memory ?

OR

Explain the concept of "Unit Root Stochastic Process". Write different particular cases of the time series $y_t = \beta_1 + \beta_2 t + \beta_3 y_{t-1} + u_t$, where u_t is a white noise error term and t is time.

2. (a) Define periodic time series. What is periodogram ? Establish a relation between periodogram and the correlogram. **7**

OR

Define Correlogram. Write its applications. Obtain the correlograms of the harmonic series.

(b) Let $X_t = y \cos \theta t + z \sin \theta t$, where y and z are two uncorrelated random variables each with mean zero and variance unity and $\theta \in (-\pi, \pi)$. In usual notations obtain γ_k and ρ_k. **7**

<div align="center">**OR**</div>

Explain the concept of ARIMA (p, d, q). Write the estimation procedure of the ARIMA model.

3. (a) Explain the concepts of Multiplier and Accelerator in Economic models. Discuss Harrod-Domar growth model (continuous case) which make use of these concepts. **7**

<div align="center">**OR**</div>

Explain 'Input-Output Analysis'. Discuss Leontief's Inter Industry Open System Model and state its important properties.

(b) Discuss Samuelson-Hicks model of multiplier accelerator. **7**

<div align="center">**OR**</div>

Discuss Mahalanobis Two Sector Model.

4. (a) Obtain correlograms for the second order autoregressive series. **7**

<div align="center">**OR**</div>

For the autoregressive series $u_{t+2} + au_{t+1} + bu_t = \varepsilon_{t+2}$ | b | < 1, show that if ε is random and the series is long then, $\dfrac{\text{Var}(u)}{\text{Var}(\varepsilon)} = \dfrac{1 + b}{(1 - b)\,((1 + b)^2 - a^2)}$. Further show that the variance of the generated series may be much greater than that of ε itself.

(b) Let $X_t = y_t + Ay_{t-1}$, where y_t's are independently and identically distributed with mean '0' and variance σ^2. Show that X_t is weakly stationary time series. **7**

<div align="center">**OR**</div>

In usual notations for the harmonic series $u_t = A \sin \theta t + \varepsilon_t$ show that $r_k = (A^2/2)$ $\cos \theta k \big(\sigma^2 + (A^2/2)\big)^{-1}$, k > 0.

5. (a) State whether the following statements are true or false. Justify your answer. **7**

   (i) The Variate Difference Method fails when cyclical component is present in the given time series.

   (ii) In the method of moving average most of the primary oscillation in the original time series will be eliminated as trend.

   (iii) Using correlogram analysis we can know the cause of oscillation in the given time series.

   (iv) The series obtained after elimination of trend will be free of oscillations.

   (v) Box Jenkins methodology is the popular name of technically known as ARIMA methodology.

   (vi) In testing the stationarity of the given time series using DF-test if the null hypothesis $H : \delta = 0$ is accepted, then the time series is non-stationary.

   (vii) The correlograms of the series of harmonic terms will oscillate but not vanish or be damped.

   (b) Answer the following questions : **7**

   (i) Define periodic time series.

   (ii) When a time series is called weak stationary ?

   (iii) Define integrated stochastic process of order d.

   (iv) Define Intensity function.

   (v) Define Unit root stochastic process.

   (vi) Define ARIMA (1, 1, 1).

   (vii) What is Buys-Ballot Table ?

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