Seat No. : _______________

AE-153

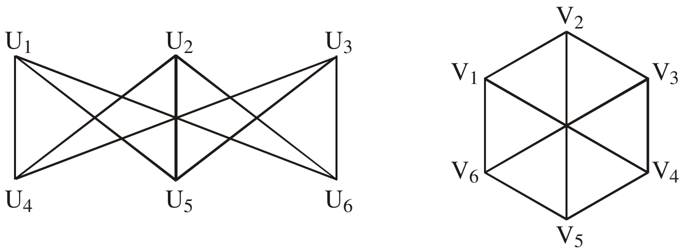
April-2015

F.Y. M.Sc., (CA & IT) Integrated
Matrix Algebra & Graph Theory

Time : 3 Hours] **[Max. Marks : 100**

1. (a) Attempt any **two** : **10**

(1) Define isomorphic graphs. Check whether the following graphs are isomorphic or not. Find the degree of each vertex in both the graphs.

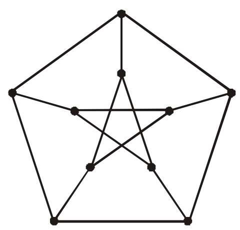


(2) Find the following for graph G :

(1) Trail of length 5

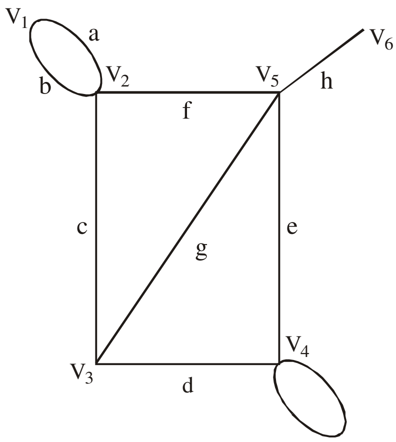
(2) Path of length 9

(3) Cycles of length 5, 6 and 8



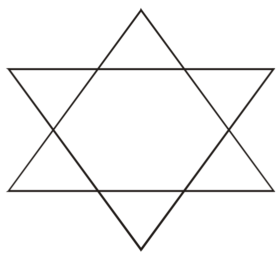
(G)

(3) Define bipartite graph and regular graph. Find adjacency matrix and incidence matrix for the following graph :

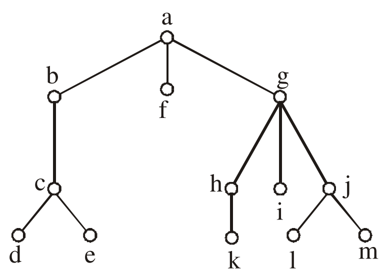


(b) Attempt any **two** : **10**

(1) Define complement of a graph. Check whether the following graph is a self complimentary or not. Also find its square.



(2) For the following tree with root a find parent of c, children of g, siblings of h, all ancestors of e, descendants of b and all internal vertices and leaves.



(3) Do as directed :

(1) Define sub graph with example.

(2) Define complete graph and draw K5.

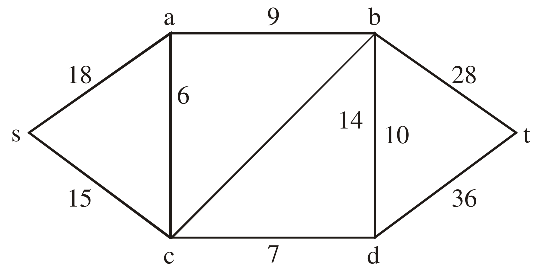
(3) Define left sub tree and right sub tree with example.

(4) How many vertices does a full 3-ary tree with 50 leaves have ?

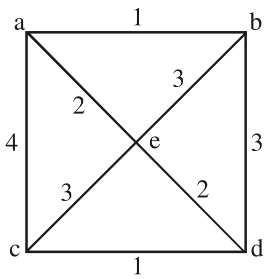
(5) How many leaves does a full 6-ary tree with 100 internal vertices have ?

2. (a) Attempt any **two** : **10**

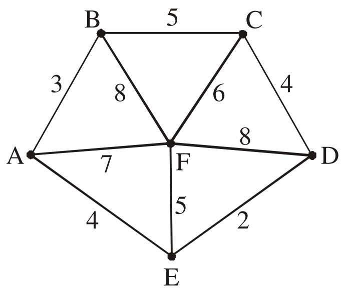
(1) Apply dijkstra's algorithm for the following weighted graph :



(2) Apply prim's algorithm for the following weighted graph :

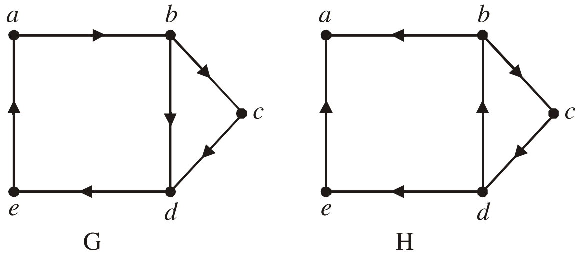


(3) Apply Kruskal's algorithm for the following weighted graph :

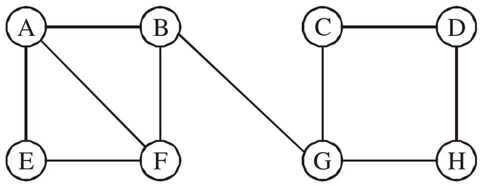


(b) Attempt **all** : **10**

(1) Define strongly connected graph, weakly connected graph and strong components. Identify whether the following graphs are strongly connected or weekly connected.



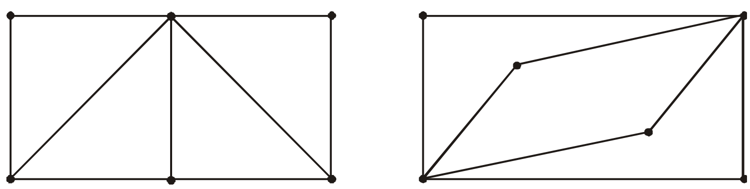
(2) Apply Breadth First search algorithm to find shortest path



3. (a) Attempt the following : **10**

(1) Define planar, non-planar and simple planar graph. Prove that complete graph k4 is planar.

(2) Define Hamiltonian path, Hamiltonian circuit and Hamiltonian graph. Check whether the following graphs are Hamiltonian graph or not.



(b) Attempt any **two** : **10**

(1) Find addition, subtraction and multiplication of matrix A and B :

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

(2) Find inverse of a matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

(3) Define diagonal matrix, upper triangular matrix and lower triangular matrix with proper example.

4. (a) Attempt the following : **10**

(1) What is Echelon matrix and Row-Reduced Echelon form ? Reduce the following matrix in the row-reduced echelon form.

$$\begin{bmatrix} 2 & 4 & -2 & 2 \\ 1 & 2 & -3 & 0 \\ 3 & 6 & -4 & 3 \end{bmatrix}$$

(2) Find rank of a given matrix :

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

(b) Attempt any **two** : **10**

(1) Use the Cayley-Hamilton theorem to find A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

(2) Test for the consistency of the following system of equations and hence solve :

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$

(3) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5. (a) Attempt the following : **10**

(1) Find the range, rank, null space and nullity of linear transformation

$T : R^3 \rightarrow R^4$ with $T(x, y, z) = (x - y + z, y - z, x, 2x - 5y + 5z)$

(2) Explain the pigeon hole principle. How many students must be in a class to guarantee that at least two students receive the same score in the final exam if the exam is graded on a scale from 0 to 100 points ?

(b) Attempt any **two** : **10**

(1) In a class of 60 students each play at least one of the three games Cricket, Hockey and Football. It is found that 24 students play Cricket, 30 students play Hockey and 24 students play Football. 6 students play both Cricket and Football. 4 students play both Hockey and Football. None of the students play three games. Find the number of students who play Cricket but not Hockey.

(2) Twenty chits marked with numbers from 1 to 20 are well mixed up. A chit is drawn at random. Find the probability that a number on the chit is (1) greater than 15 and (2) divisible by 3 and 5.

(3) (1) How many different permutations can be made out of the letters of word **COMPUTER** ?

(2) Expand $(x + y)^n$

(3) A group of 30 people have been trained as astronauts to go on the mission to Mars. How many ways are there to select a crew of six people to go on this mission ?