4/31 **3010E941** Candidate's Seat No :________

B.Sc. Sem.-5 (Rep) Examination

CC-301

Mathematics (New)

October-2025

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions:

(i) All the questions are compulsory and carry equal marks.
(ii) Notations are usual everywhere.
(iii) The right-hand side figures indicate marks of the question/sub-question.

Q-1 (a) Explain the Bisection method for the solution of algebraic and transcendental equations. [7]

(b) Find the maximum relative error in $u = \frac{xy^2}{z^3}$ at x = y = z = 1 when the error in each of x, y, z is 0.001 [7]

OR

Q-1(a) Explain the False Position method for the solution of algebraic and transcendental equations. Also find relative error in the result. [7]

(*b*) Find a real root of $f(x) = x^3 + x^2 + 2x - 1 = 0$ correct upto three decimal places by the Newton-Raphson's method. [7]

Q-2(a) What is interpolation?
Derive Newton's forward interpolation formula and explain when it is used. [7]

(b) In usual notations, prove that $\Delta^n = u_x - nu_{x-1} + \frac{n(n-1)}{2}u_{x-2} + \cdots + (-1)^n u_{x-n}$. [7]

OR

Q-2(a) Define averaging operator μ and central difference operator δ. Also derive the relation $\mu^2 = 1 + \frac{1}{4}\delta^2$. [7]

(b) Find the missing terms in the following table: [7]

x	0	1	2	3	4
y	1	3	9	?	81

Q-3(a) Establish Newton's divided difference interpolation formula. [7]

(b) State Lagrange's interpolation formula and using it find Lagrange interpolating polynomial of degree 2 approximating the function y = ln x defined by

y(2) = 0.69315, y(2.5) = 0.91629, y(3) = 1.09861.

Also determine the value of ln 2.7. [7]

OR

Q-3(a) Prove that divided differences are symmetrical in their argument.

Also, prove that n^{th} divided difference of n^{th} degree polynomial is constant [7]

(b) Find x for which y is maximum, correct to two decimal places from the following table. Also, find the maximum value of y. [7]

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996

Q-4(a) Obtain a general formula for numerical integration using Newton's Forward Interpolation Formula.

And derive Simpson's $\frac{3}{8}$ Rule from it. [7]

(b) Evaluate integral $I = \int_0^{\frac{\pi}{2}} \sin x \, dx$ by using (i) Simpson's $\frac{1}{3}rd$ Rule and (ii) Trapezoidal Rule. [7]

OR

Q-4(a) Using Range-Kutta forth order Formula solve $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$

at the points 0.2 and 0.4. [7]

(b) Solve the differential equation $\frac{dy}{dx} = x + y, y(0) = 0$ by using Euler's Modified method.

Choose $h = 0.2$ and compute $y(0.2)$ and $y(0.4)$. [7]

Q-5 Attempt any SEVEN in short: [14]

(*a*) Find the relative error of the number 7.3 if both of its digits are correct.
(*b*) Find the relative error of the number 8.6 if both of its digits are correct.
(*c*) What is percentage off error? Give example.
(*d*) What is round off error? Give example.
(*e*) Write down formula to find out x_1, the first approximation to the root of $f(x) = 0$ in interval $[a, b]$.
(*f*) Prove or disprove: The bisection method does not guarantee convergence.
(*g*) In usual notations, prove that $\nabla = \delta E^{-1} = \delta E^{1/2}$
(*h*) Find $\Delta[(x + 3)(x + 4)]$.
(*i*) Express $\Delta^2 y_{-1}$ in terms of $\Delta^2 y_0$, $\Delta^3 y_0$, ...
(*j*) What is the degree of a polynomial required for interpolation given n+1 data points?
(*k*) What is the purpose of numerical differentiation?
(*l*) Derive the expression for $[x_1, x_2, x_3]$ in terms of Δ.

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3010E941-3 Candidate's Seat No : ________

B.Sc. Sem.-5 (Rep) Examination
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Time : 2-30 Hours] **[Max. Marks : 70**

Que.1 (A) Define an annihilator. Prove that if A is a subset of real vector space V then A^0 (annihilator of A) is a subspace of V^*. [7]

(B) Find the Dual basis of the basis $B_2 = \{(1,-1,3),(0,1,-1),(0,3,-2)\}$ for the vector space R^3. [7]

OR

Que.1 (A) Prove that the addition of two linear maps is a linear map. [7]

(B) If a linear map $T : R^3 \to R^2$ is definded as $T(x,y,z) = (x+y, 2y+z)$; $(x,y,z) \in R^3$,

Then solve the operator equation $T(x,y,z) = (2,4)$. [7]

Que.2 (A) State and prove the Cauchy-schwarz in equality. [7]

(B) Apply the Gram-Schmidt orthgonalization process to the basis $B_2 = \{(1,-1,3),(0,1,-1),(0,3,-2)\}$ in order to get orthogonal and orthonormal basis for vector space R^3. [7]

OR

Que.2 (A) Prove that every finite dimensional inner product space has an orthonormal basis. [7]

(B) If for $X = (x_1, x_2)$, $Y = (y_1, y_2) \in R^2$ then the map $\prec , \succ$ is defined as $\prec X, Y \succ = \frac{1}{4}(x_1 - x_2)(y_1 - y_2) + \frac{1}{4}(x_1 + x_2)(y_1 + y_2)$ then show that $\prec , \succ$ is an inner product on R^2. [7]

Que.3 (A) For matrix $A = (a_{ij}) \in M_n$, Prove that

$$\det A = \sum_{f \in S_n} (\operatorname{sgn} f) a_{f(1)1} a_{f(2)2}, \ldots, a_{f(n)n}.$$ [7]

(B) State (only) the Cramer's rule and using it Solve $2x + y = 2$, $3y + z = 1$ and $4z + x = 5$. [7]

OR

Que.3 (A) In usual notation, Prove that $\det(AB) = \det A \cdot \det B$. [7]

(B) If $f = \begin{pmatrix} 1234567 \\ 3451726 \end{pmatrix}$; $g = \begin{pmatrix} 1234567 \\ 2546173 \end{pmatrix} \in S_7$, then

Find fog, $(gof)^{-1}$, $\operatorname{sgn} f$ and $\operatorname{sgn}(g^{-1})$. [7]

P.T.O

Que.4 (A) Prove that the eigen values of symmetric linear transformation are real. [7]

(B) Diagonalize the matrix $A=\begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 04 & -2 & -4 \end{pmatrix}$. [7]

OR

Que.4 (A) Prove that distinct eigen vectors of T $\in$ L(U,V) co-rresponding to distinct eigen values of T are linearly independent. [7]

(B) Verify the Caley-Hamilton theorem for the matrix $A=\begin{bmatrix} 3 & 4 & 3 \\ 4 & 6 & 2 \\ 3 & 2 & 11 \end{bmatrix}$. [7]

Que.5 Answer the following question in short : (Any Seven) [14]

(1) Define the Dual basis.

(2) Find $N(T)$ for the linear transformation $T: R^4 \to R^3$ defined by $T(x,y,z,w)=(x-w, y+z, z-w)$.

(3) What is the dimension of vector space $L(R^2, R^4)$ and $L(R, R^2)$?

(4) In inner product space V, prove that $\|x\|=\|y\|$ if and only if $(x-y)\perp(x+y)$.

(5) Find a non-zero vector orthogonal to the vector $x=(1,2,1)$ and $y=(4,5,2)$ in an inner product space R^3 with standard inner product.

(6) State and prove triangle inequality in an inner product space V.

(7) Define similar matrices.

(8) If $\det A=-2$ then find $\det(A^3)$.

(9) State laplace expansion.

(10) Define eigen values and eigen vectors of a linear operator.

(11) Define symmetric linear transformation.

(12) write the matrix form of $10x^2+2xy+7y^2=100$.

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