2910E925

Candidate's Seat No :________

M.Sc. Sem.-4 (Rep) Examination

507 - Mathematics

October-2025

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Show that the tangents to a helix $f(t) = (\cos\omega t,\ \sin\omega t,\ t)$ make a constant angle with the $xy-$plane. 7

   (B) Show that if the tangents to a curve are parallel to a certain plane then the curve is a plane curve. 7

OR

1. (A) Define regular curve with illustrations. 7

   (B) Find the curvature and torsion of the curve $x = 2\cos t,\ y = 2\sin t,\ z = 0.$ 7

2. (A) Define the osculating paraboloid of a surface at a point. Find the equation of the osculating paraboloid to an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $(0, 0, c)$, $c > 0$. 7

   (B) Prove that if all normals to a surface are concurrent, then the surface is either a sphere or a spherical domain. 7

OR

2. (A) Define the equation of the tangent plane to a surface at a point. Find the equation of the tangent plane to a sphere $x = \cos u \cos v,\ y = \cos u \sin v,\ z = \sin u$ at the point (1, 0, 0). 7

   (B) Define ellptic, hyperbolic and planar points.

3. (A) Define the first fundamental form of a surface $\overline{r}(u, v)$. Define u-curves and v-curves on the surface $\overline{r}(u, v)$. 7

   (B) Define Gaussian curvature and mean curvature. Show that the Gaussian curvature of a hyperbolic paraboloid $z = 2xy$ at the point $(0, 0, 0)$ is negative and the mean curvature is zero. 7

OR

3. (A) Define (i) geodesic (line) and (ii) a line of curvature on a surface.
   Show that if a geodesic is a line of curvature then it lies in a plane. 7

   (B) Define Gaussian curvature and mean curvature. Show that the Gaussian curvature of a sphere of radius r is $\frac{1}{r^2}$. 7

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4. (A) Show that the surface area of a surface ϕ is given by $\iint_{\phi} \sqrt{EG - F^2}$. **7**

(B) Prove that the area of any geodesic triangle on a sphere is greater than π. **7**

OR

4. (A) State (without proof) Gauss Bonnet theorem. **7**

(B) Prove that the sum of the angles of a geodesic triangle on a surface of positive Gaussian curvature is greater than π, and less than π on a surface of negative curvature. **7**

5. **Attempt any SEVEN of the following:** **14**

(1) What is the curvature and torsion of the curve $\overline{\alpha}(t) = (3\cos t, 3\sin t)$(respectively)?

(A) 3, 0
(B) 0, 3
(C) 1/3, 0
(D) 0, 1/3

(2) Identify the curve given by $\alpha(t) = (t,\ \cosh t)$.

(A) cycloid
(B) catenary
(C) astroid
(D) tractrix

(3) Which one of the following curves lies on a plane?

(A) $x = t,\ y = 0,\ z = t^3$
(B) $x = t,\ y = t^3, z = t^4$
(C) $x = \cos t,\ y = \sin t,\ z = 3\sin t + 4\cos t$
(D) $x = \cos t,\ y = \sin t,\ z = t$

(4) The form of the surface given by $z = xy$ is ________

(A) an ellipsoid
(B) a plane
(C) a hyperboloid of one sheet
(D) a hyperbolic paraboloid

(5) If $\overline{\alpha}(s)$ is a regular curve with the curvature $\kappa_1(s) = 0$ at each point then the curve $\overline{\alpha}(s)$ is ________

(A) a circle
(B) plane curve
(C) straight line
(D) none of the above

(6) The equation of the osculating paraboloid to the ellipsoid $\frac{x^2}{2^2} + \frac{y^2}{3^2} + \frac{z^2}{2^2} = 1$ at the point $(0, 0, 2)$ is $z =$ ________

(A) $1 - \frac{1}{2}(x^2 + x^3 + y)$
(B) $2(1 - \frac{x^2}{8} - \frac{y^2}{18})$
(C) $1 + \frac{1}{2}(x^2 + x^3 + y^2)$
(D) $1 + x^2 + y^2$

(7) Every point of the surface $z = 4x^2 + 4y^2$ is

(A) planar
(B) parabolic
(C) hyperbolic
(D) elliptic

(8) Which of the following are regular closed surfaces?

(A) A sphere
(B) A cone
(C) A torus
(D) A sphere with a deleted meridian.

(9) The sum of all the three interior angles of a geodesic triangle on a surface with Gaussian curvature zero is ________

(A) equal to π
(B) greater than π
(C) less than π
(D) equal to 2π

(10) What is the Euler's characteristic of the torus ?

(A) 1
(B) 2
(C) 3
(D) 0

(11) The shortest path on the surface $x^2 + y^2 + z^2 = 4$, between the points $(1, 1, \sqrt{2})$ and $(-1, -1, -\sqrt{2})$ has length

(A) 2 (B) 4 (C) 2π (D) $5\sqrt{2}$

(12) Which of the following surfaces does not have constant Gaussian curvature?

(A) A surface of revolution
(B) Pseudo sphere
(C) A plane
(D) A torus
