2910E936/A Candidate's Seat No :________

Intg. M.Sc. (DS) Sem.-3 (Supply.) Examination

CC-201 - Matrix Algebra and Calculus

Time : 2-30 Hours] **October-2025** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the vector projection of $u = 6i + 3j + 2k$ onto $v = i - 2j - 2k$ and the scalar component of u in the direction of v. **(07)**

(b) **i.** Prove that the angle subtended between $(1, 1, -1)$ and $(2, -2, 1)$ is $sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$. **(07)**

ii. Find the unit vector perpendicular to the plane containing the vectors $a = i - j + k$ and $b = 2i + 3j - k$.

OR

(a) A particle is displaced from the point $(0, 1, -2)$ to the point $(-1, 3, 2)$ under the action of applied forces $(1, 2, 3), (-1, 2, 3)$ and $(-1, 2, -3)$, then find the work done by the particle. **(07)**

(b) A force $F = 2i + j - 3k$ is applied to a spacecraft with velocity vector $v = 3i - j$. Express F as a sum of a vector parallel to v and a vector orthogonal to v. **(07)**

Q.2 **(a)** For which value of k and μ the following system have (i) No solution (ii) Unique solution (iii) an infinite number of solutions **(07)**

$$x + y + z = 6, x + 2y + 3z = 10, x + 2y + kz = \mu$$

(b) Define the rank of the matrix. Find the rank of the matrix $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ **(07)**

OR

(a) Show that $A = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is orthogonal. **(07)**

(b) Find the inverse of matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ using Gauss-Jordan Method **(07)**

Q.3 **(a)** Solve the following system of equations using partial pivoting by the Gauss Elimination method: **(07)**

$$2x_1 + 2x_2 + x_3 = 6, \quad 4x_1 + 2x_2 + 3x_3 = 4, \quad x_1 + x_2 + x_3 = 0$$

(b) Obtain Cholesky factorization of the following matrix: **(07)**

$$\begin{bmatrix} 4 & 8 & 12 \\ 8 & 20 & 20 \\ 12 & 20 & 41 \end{bmatrix}$$

OR

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(a) Solve the following system of linear equation by using Gauss-Siedel method: (07)

$$10x + y + z = 6, x + 10y + z = 6, x + y + 10z = 6$$

(b) Solve the following system of linear equations by using Crout's method: (07)

$$10x + y + z = 12, \quad 2x + 10y + z = 13, \quad 2x + 2y + 10z = 14$$

Q.4 (a) Suppose that the Celsius temperature at the point (x, y, z) on the sphere $x^2 + y^2 + z^2 = 1$ is $T = 400xyz^2$. Locate the highest and lowest temperature on the sphere. (07)

(b) Find the Taylor's series formula for $f(x, y)$ at the origin to the given function $f(x, y) = xcosy$ up to two terms. (07)

OR

(a) Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2, x = r - s, y = r + s.$ (07)

(b) Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 - z^2 = 18$ at the point $P(3,5,-4)$ (07)

Q.5 Attempt any **SEVEN** out of **TWELVE:** (14)

(1) Find the unit vector perpendicular to the plane containing the vectors $a = i - j + k$ and $b = 2i + 3j - k$.

(2) Define: Direction Cosines of the vector

(3) Explain Ill-conditioned and well-conditioned system with suitable example.

(4) What is trivial solution of Homogeneous system of equations? Explain with suitable example.

(5) State necessary condition for the existence of Gauss-Jacobi method.

(6) Define Symmetric and Skew Symmetric matrix.

(7) Check whether the following system is diagonally dominant or not? Justify your answer.

$$10x - 4y + z = 7, x + 5y - 2z = 5, 8x - 4y - 3z = 6$$

(8) Investigate for what values of k the equations

$$x + y + z = 1, \quad 2x + y + 4z = k, \quad 4x + y + 10z = k^2$$

have infinite number of solutions.

(9) Find domain and range for the function: $z = \sqrt{x^2 - y}$

(10) Find the limit of the function: $\lim_{(x,y)\to(1,2)} \frac{x^3+y^3}{xy^2+x^3}$

(11) Define: Continuity of two variable at a point

(12) Define: Chain Rule of the function $z = f(x, y)$

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