Seat No. : _______________

AC-113

April-2015

B.Sc., Sem.-VI

MAT-307

Abstract Algebra-II

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry **14** marks.
(2) Figures to the right indicate marks of the question/sub question.
(3) Notations are as usual.

1. (a) Define the following terms with examples : **7**
   (i) Division Ring
   (ii) Integral Domain
   (iii) Field

   OR

   Prove that every finite integral domain is a field.

   (b) Prove that each non-zero element of $F = \{a + b\sqrt{2} \,/ a,b \in \mathbb{Q}\}$ is a unit element. **7**

   OR

   Define characteristic of a ring.
   Prove that the characteristic of an integral domain is either prime number or zero.

2. (a) Define 'Principal ideal'. **7**
   Prove that the ring (Z, +, .) of all integers is the principal ideal ring.

   OR

   Define ideal in a ring. Give an example of :
   (i) Left ideal which is not right ideal.
   (ii) Right ideal which is not left ideal.
   (iii) Both sided ideal.

   (b) Define Kernel of a ring homomorphism. **7**
   If f is a homomorphism of ring R into a ring R' with kernel K, then prove that K is an ideal of ring R.

   OR

   Prove that a field has no proper ideal.

3. (a) Find the G.C.D. of polynomials. **7**

$f(x) = x^3 - 3x^2 + 2x - 6$ and $g(x) = x^3 - 4x^2 + 4x - 3$ over the field R. Express the G.C.D. as a linear combination of two polynomials.

OR

Define degree of a polynomial in D[x].

If f, g $\in$ D [x] are nonzero polynomials, then prove that deg (fg) = deg (f) + deg(g).

(b) State and prove Remainder theorem. **7**

Also find all zeroes of $x^4 + 3x^3 + 2x + 4 \in Z_5[x]$ in Z_5.

OR

State and prove Division algorithm for polynomials.

4. (a) An ideal I in a commutative ring R with unity is a prime ideal iff the quotient ring R / I is an integral domain. **7**

OR

An ideal I in a commutative ring R with unity is maximal ideal iff the quotient ring R/I is a field.

(b) Give an example of a ring in which some prime ideal is not a maximal ideal. **7**

OR

Prove that field $Q[x]/ < x^2 - 2 >$ is isomorphic to field $\mathbb{Q}(\sqrt{2})$.

5. Answer in short : (any **seven**) **14**

(i) Define 'unit element' in a ring with unity.

(ii) Is the ring $(\mathbb{Z}_6, +_6, \times_6)$ an integral domain ?

(iii) Prove that Boolean ring is a commutative ring.

(iv) Give an example of a subring which is not an ideal in some ring.

(v) Is the ideal generated by single element a principal ideal ? What is ideal generated by 1 in the ring (Z, +, .) ?

(vi) Is the polynomial $f(x) = x^2 + 9$ reducible in $\mathbb{Q}[x]$? Also check its reducibility in $\mathbb{C}[x]$.

(vii) Define associate polynomials.

(viii) State Eisenstein criterion for irreducibility.

(ix) Is the ideal $\langle 4 \rangle$ maximal ideal in the ring of integers ? Justify your answer.

(x) List all possible rational zeroes of $f(x) = 4x^5 + x^3 + x^2 - 3x + 1$.