0311E999 Candidate's Seat No :__

B.Sc. Sem.-5 Examination
CC - 303
Mathematics
November-2025

Time : 2-30 Hours] **[Max. Ma**

Instructions : (1) All questions are compulsory.
(2) Write the question number in your answer sheet as shown in the question paper.
(3) Figures to the right indicate marks of the question.

Q-1(a)(i) State and prove any seven properties of complex numbers.

(ii) Define trigonometric and hyperbolic functions in $\mathbb{C}$. Solve $\sin z = 2$.

OR

Q-1(a)(i) State and prove De Moivre's theorem. Hence solve $z^3 = 8i$.

(ii) Define convergence of the series of complex numbers. Suppose that $z_n = x_n + iy_n, n\epsilon\mathbb{N}$ and $S = X + iY$, then prove that $\sum_{n=1}^{\infty} z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

Q-2(a)(i) State and prove necessary conditions for a function $f(x,y) = u(x,y) + iv(x,y)$ to be analytic at a point in a region R.

(ii) Show that $f(z) = e^{2z}$ is an entire function and also find its derivative.

OR

Q-2(a)(i) If $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} ; z \neq 0 \\ 0 \quad ; z = 0 \end{cases}$ then, check whether component functions satisfy C-R equations at origin. Also check if $f(z)$ is analytic at origin.

(ii) If $f(x,y) = u(x,y) + iv(x,y)$ is analytic in a domain D with constant modulus, then prove that the function f is constant. Also find harmonic conjugate of the function $u(x,y) = x^2 - y^2$.

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Q-3(a)(i) If $f(z)$ is analytic at a point z_0 of the domain then prove that mapping $w = f(z)$ is conformal at z_0 iff $f'(z_0) \neq 0$.

(ii) Find the image of curves given by

(i) $A(x^2 + y^2) + Bx + Cy + D = 0$,

(ii) $D(u^2 + v^2) + Bu - Cv + A = 0$ and

(iii) a line $x = C$ (A, B, C and D are constants)

under $w = 1/z$.

OR

Q-3(a)(i) Define Linear fractional transformation. Find a Linear fractional transformation that maps the points $z_1 = 0,\ z_2 = i\ and\ z_3 = -i$ onto the points $w_1 = -1,\ w_2 = \infty\ and\ w_3 = 0$.

(ii) Consider the map $w = (1 + i)z$. Find image R' in w plane of the rectangular region R bounded by the lines $x = 0, y = 0, x = 4$ and $y = 4$ in z plane. Sketch both R and R'.

Q-4(a)(i) State and prove Bessel's inequality.

(ii) Find Fourier series expansion of $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$.

OR

Q-4(a)(i) State and prove Riemann-Lebesgue theorem.

(ii) Find Fourier series expansion of $f(x) = x^3, -1 < x < 1$.

Q-5 Give the answer in brief. (Any Seven).

1) Plot all the roots of $z^5 = 1$.

2) State triangle inequality.

3) Evaluate $ln\ (1 + i)$.

4) Define an Entire function giving one example.

5) Define harmonic conjugate.

6) Can there be a function which is not analytic at exactly three points? If so give one example (without justification).

7) Find points where $f(z) = \sin z$ is conformal.

8) Define Bilinear transformation.

9) Find fixed points of $w = \frac{z+1}{4z-3}, z \neq \frac{3}{4}$

10) Can there be a Periodic function which is not continuous? If so give one example (without justification).

11) Find Fourier coefficient a_0 for the function $f(x) = x^2$ in $(-2,2)$.

12) Find Fourier series expansion for the function $f(x) = \sin x$ in $(0,2\pi)$.

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