0111E975

Candidate's Seat No :________

M.Sc. Sem.-4 (Rep) Examination

509 - Mathematics (EA)

N.T.

Time : 2-30 Hours] **November-2025** **[Max. Marks : 70**

1. (A) Define Möbius function. Show that Möbius function is multiplicative. 7

   (B) Determine all solutions in the positive integers of the equation $18x + 5y = 48$. 7

OR

   (A) Define Fermat number and show that Fermat numbers are all relatively prime to each other. 7

   (B) (i) Find the highest power of 12 contained in 400!.

   (ii) Find the highest power of 3 which divides 31!. 7

2. (A) State and prove Chinese remainder theorem. 7

   (B) Find the remainder when 72^{1001} is divided by 31. 7

OR

   (A) State and prove Euler's theorem. 7

   (B) Solve the sets of following simultaneous congruence 7

$$x \equiv 2(mod\ 3),\ x \equiv 3(mod\ 5),\ x \equiv 2(mod\ 7).$$

3. (A) For odd prime p, prove that there are precisely $\frac{(p-1)}{2}$ quadratic residues and $\frac{(p-1)}{2}$ quadratic nonresidues of p. 7

   (B) State and prove Euler's criterion for quadratic residue. 7

OR

   (A) Prove that there are infinitely many primes of the form $8k - 1$. 7

   (B) Solve the congruence $4x^9 \equiv 7(mod\ 13)$. 7

4. (A) Prove that rational number can be written as a finite simple continued fraction. 7

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(B) Obtain all primitive Pythagorean triples x, y, z in which $x = 60$. **7**

OR

(A) Define Pythagorean triangle. Prove that the radius of the inscribed circle of a Pythagorean triangle is always an integer. **7**

(B) By means of continued fractions determine the general solutions of Diophantine equation $172x + 20y = 1000$. **7**

5. **Attempt any seven of the following.** 14

(1) Find the highest power of 2 which divides 25!.

(2) Find the number of positive divisors of 2025.

(3) Find the value of Möbius function $\mu(2025)$.

(4) Does the number 28 a perfect number?

(5) Find the remainder when 5^{38} is divided by 11.

(6) Find the order of 3 modulo 19.

(7) Is the equation $x^2 \equiv 8 (mod\, 13)$ solvable? Justify.

(8) Find the index of 3 relative to primitive root 6 of 11.

(9) Does the number 36 has primitive root? Justify.

(10) Find 3^{rd} convergent of finite simple continued fraction $[0; 2, 1, 2, 6]$.

(11) Express the rational number $\frac{71}{51}$ as finite simple continued fraction.

(12) Find the value of finite continued fraction $[0; 1, 2, 3]$.
