

## B.Sc. Sem.-5 (Rep) Examination

## CC - 302 - Mathematics

November-2025

[Max. Marks : 70]

Time : 2-30 Hours]

**Instructions**

- There are total five questions.
- All questions are compulsory.
- Figures on the right side show the marks.
- Notations and terminologies are standard.

- Q.1 (a) State and prove rational density theorem. [7]  
 (b) Prove that  $\sqrt{2}$  is not rational. [7]

OR

- Q.1 (a) State and prove the Archimedean property. Find  $glb$  of  $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ , if exists. [7]  
 (b) Prove that the set  $A = \{3n : n \in \mathbb{N}\}$  is denumerable set. [7]
- Q.2 (a) Prove that every bounded sequence has a convergent sequences. [7]  
 (b) Discuss the convergence of the sequence  $\{\frac{n}{3} - [\frac{n}{3}]\}$  [7]

OR

- Q.2 (a) State and prove Cauchy's completeness principle. [7]  
 (b) Prove that the sequence  $\{\sum_{k=1}^n \frac{1}{k}\}$  is not Cauchy sequence using definition. [7]
- Q.3 (a) State and prove intermediate value theorem. [7]  
 (b) Discuss uniform continuity of the function  $f(x) = \sin(\frac{2\pi}{x})$  on  $(0, 1)$ . [7]

OR

- Q.3 (a) Let  $\lim_{x \rightarrow c} f(x) = L_1$  and  $\lim_{x \rightarrow c} g(x) = L_2$ . Then prove that  $\lim_{x \rightarrow c} f(x) \cdot g(x) = L_1 \cdot L_2$ . [7]  
 (b) Prove that any polynomial of odd degree must have at least one real root. [7]
- Q.4 (a) State and prove inverse function theorem. [7]  
 (b) Determine the extreme values of  $f(x) = |x^2 - 4|$  in  $[-2, 2]$ , if exist. [7]

OR

- Q.4 (a) State and prove Darboux's Mean Value Theorem. [7]  
 (b) Let  $n \in \mathbb{N}$  be an odd and define  $f(x) = x^n$  for all  $x \in \mathbb{R}$ . Show that  $f$  is strictly increasing and one-to-one on  $\mathbb{R}$ . Is  $f^{-1}$  differentiable? if yes, find it's derivative. [7]

Q.5 Attempt any seven. *in short*;

[14]

- (1) By definition prove that  $\text{lub} \{1 - \frac{1}{n+1} : n \in \mathbb{N}\} = 1$ .
- (2) Define: Ordered field. Is the set of complex number  $\mathbb{C}$  an ordered field?
- (3) State True or False: The set of all transcendental numbers is countable set.
- (4) Give an example of a sequence which is bounded but not convergent.
- (5) Find the set of cluster points of sequence  $a_n = \sin \frac{n\pi}{4}$ .
- (6) Give an example of an unbounded sequence which has a convergent subsequence.
- (7) Find the  $\lim_{x \rightarrow 5} \sin(\pi[\frac{x}{2}])$ , if exists.
- (8) Define: Uniform continuity.
- (9) State extreme Value Theorem.
- (10) Is mean value theorem applicable to  $f(x) = [x]$  on  $[1, 2]$ ? Justify.
- (11) If  $f(0) = f'(0) = 1$ , then evaluate  $\lim_{x \rightarrow 0} \frac{f(x)-1}{x}$ .
- (12) State Roll's Mean Value Theorem.

~~X~~ — ~~X~~