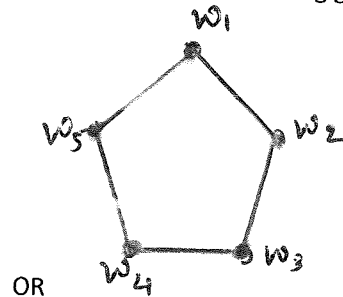
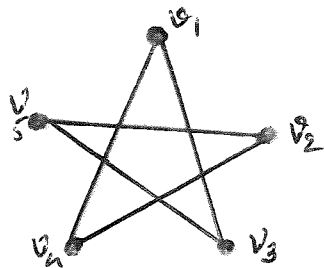


**Instructions:**

- (a) Notations and terminologies are standard.
- (b) All the questions are compulsory.
- (c) Each question carries equal marks.

Q – 1

- (a) State the First Theorem of Graph Theory and prove that in any graph  $G$ , there is an even number of odd vertices. 7
- (b) Define Isomorphism of a graph and check whether the following graphs are isomorphic or not. 7

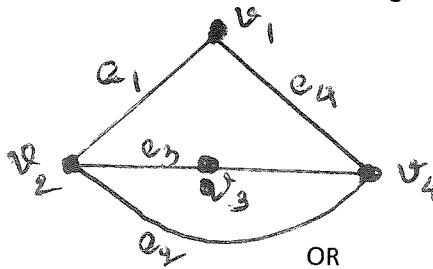


OR

- (a) Define the following terms. 7
  - (i) Subgraph
  - (ii) Proper Subgraph
  - (iii) Spanning Subgraph
  - (vi) Edge deleted subgraph
- (b) If  $G$  be a  $k$ -regular graph, where  $k$  is an odd number, then prove that the number of edges in  $G$  is a multiple of  $k$ . 7

Q – 2

- (a) Prove that if  $G$  is a nonempty graph with at least two vertices, then  $G$  is bipartite if and only if it has no odd cycles. 7
- (b) Find Adjacency and incidence matrix of the given graph. 7



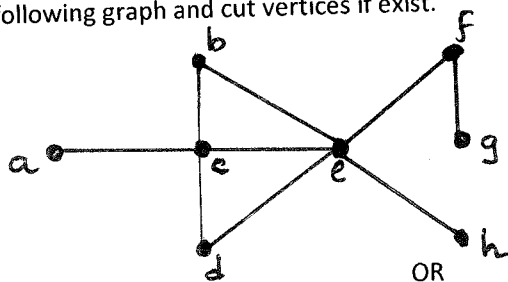
OR

- (a) If  $T$  is a tree, then prove that it has precisely  $n - 1$  edges. 7
- (b) If  $G$  is a graph with  $n$  vertices,  $q$  edges and  $w(G)$ , which is the number of connected components of  $G$ , then prove that  $q \geq n - w(G)$ . 7

Q-3

(a) If  $G$  is a graph with  $n$  vertices, where  $n \geq 2$ . Then prove that  $G$  has at least two vertices, which are not cut vertices. 7

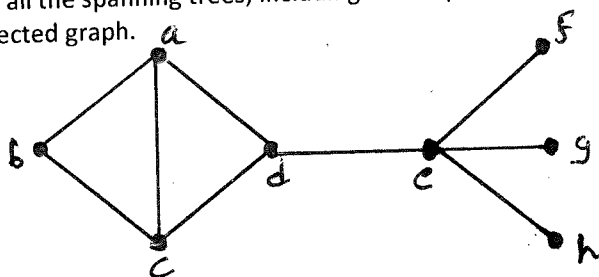
(b) Define cut vertex and connectivity of a graph. Find connectivity  $k(G)$  of the following graph and cut vertices if exist. 7



OR

(a) Prove that a graph  $G$  is connected if and only if it has a spanning tree. 7

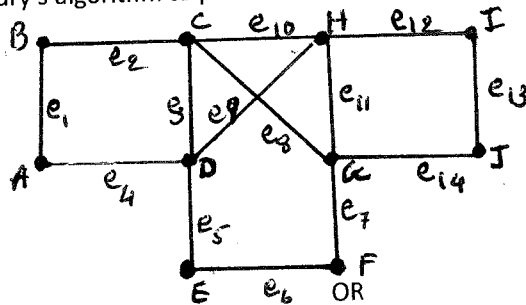
(b) Draw all the spanning trees, including isomorphic ones, of the following connected graph. 7



Q-4

(a) If  $G$  be a graph in which the degree of every vertex is at least two, then prove that  $G$  contains a cycle. 7

(b) Use Fleury's algorithm to produce an Euler tour for the following graph. 7



(a) Prove that a connected graph  $G$  is Euler if and only if the degree of every vertex is even. 7

(b) If  $G$  is a simple graph with  $n$  vertices, where  $n \geq 3$  and the degree  $d(v) \geq \frac{n}{2}$  for every vertex  $v$  of  $G$ , then prove that  $G$  is Hamiltonian. 7

Q-5

- Attempt Any Seven. *in short* 14
- (i) Define bipartite and complete bipartite graph.
  - (ii) Give an example of a self-complement graph and justify your answer.
  - (iii) Draw a graph with 4 vertices, which is 2-regular.
  - (iv) Define Tree and Forest.
  - (v) Find radius and diameter of complete graph  $K_5$ .

(vi) Draw a graph, whose adjacency matrix is  $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

N 703.3

- (vii) State Cayley's theorem.
- (viii) Find the connectivity of  $K_5$  and  $K_1$ .
- (ix) State Whitney's theorem.
- (x) Check whether complete graph  $K_4$  is Euler or not.
- (xi) Define Hamiltonian graph.
- (xii) Define closure of graph  $G$ .

