

B.Sc. Sem.-6 (Rep) Examination

CC-309

Mathematics

October-2025

Time : 2-30 Hours]

[Max. Marks : 70

Instruction: (i) All questions are compulsory.

(ii) Right hand side figures indicate marks of the question

- Q-1 (A) Let X be a metric space. Prove that an open sphere is an open set. [7]
 (B) Let X be a metric space. A subset F of X is closed if and only if its complement F' is open. [7]

OR

- Q-1 (A) Let (X, d) be a complete metric space and Y be a subspace of X . Then prove that Y is complete if and only if it is closed in (X, d) . [7]
 (B) In a metric space X prove that every closed sphere is a closed set. [7]

- Q-2 (A) Prove that compact subsets of metric spaces are closed. [7]
 (B) Let X and Y be metric spaces and f a mapping of X into Y then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y . [7]

OR

- Q-2 (A) A subset E of the real line R_1 is connected if and only if it has the following property : if $x \in E, y \in E$ and $x < z < y$ then $z \in E$ [7]
 (B) Let (X, d_1) and (Y, d_2) be metric spaces then prove that $f : X \rightarrow Y$ is continuous on X iff $f^{-1}(F)$ is closed in X , whenever F is closed in Y . [7]

- Q-3 (A) State and prove Weierstrass M_n -test. Show that the sequence $\{f_n(x)\}$ is not uniformly convergent on any interval containing zero where $f_n(x) = \frac{nx}{1+n^2x^2}$ [7]

- (B) Let (f_n) be a sequence of functions in $R[a, b]$ converging uniformly to f . Then $f \in R[a, b]$ and $\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = \int_a^b f(x) dx$. [7]

OR

- Q-3 (A) Let (f_n) be a sequence of continuous function on $E \subset C$ converges uniformly to f on E then prove that f is continuous on E . [7]
 (B) Let f_n satisfy (1) $f_n \in D[a, b]$ (2) $(f_n(x_0))$ converges for $x_0 \in D[a, b]$ (3) f_n' converges uniformly on $[a, b]$ then prove that f_n converges uniformly on $[a, b]$ to a function f . [7]

- Q-4 (A) State and prove Abel's limit theorem. [7]
 (B) For every $x \in R$, and $n > 0$, prove that $\sum_{k=0}^n (nx-k)^2 \binom{n}{k} x^k (1-x)^{n-k} = nx(1-x) \leq \frac{n}{4}$ [7]

OR

- Q-4 (A) State and prove Weierstrass Approximation theorem. [7]
 (B) State Taylor's series.

Show that for $-1 \leq x \leq 1$, $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$. Hence evaluate $\log 2$. [7]

(P.T.O...)

N 686-2

Q-5 Answer in Short (Any SEVEN)

[14]

Define a discrete metric on a set X .

1.

then prove that X is a metric space.

2. Define metric space.

3. Define Derive set. Give one example of derive set.

4. Define compact set.

5. Define finite subcover.

6. Prove that open interval $(0,1)$ with usual metric is not compact.

7. Is $f_n(x) = \frac{1}{1+nx}$ ($x \geq 0$) continuous? justify.

8. If the series $\sum a_k$ converges absolutely then prove that the series $\sum a_k \cos kx$ is uniformly convergent on \mathbb{R} .

9. Show that the series $\sum_{k=0}^{\infty} (xe^{-x})^k$ is uniformly convergent.

10. Prove by Taylor's series $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

11. State Bernstein Theorem.

12. For every $x \in \mathbb{R}$ and $n \geq 0$ prove that $\sum_{k=0}^n \binom{n}{k} x^k (1-x)^{n-k} = 1$

-----Best of Luck-----