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1305E614

Candidate's Seat No:_____

IMSc AIML (NEP) Sem.2 Examination

MDC- AIML-124

Mathematical Science Concept-II

Time : 2.00 Hours] **May-2025** **[Max. Marks : 50**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Using Vertically and Crosswise Sutra, solve the determinant: **(05)**

$$D = \begin{vmatrix} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{vmatrix}$$

(b) Using Urdhva Tiryag sutra, solve the integral of $\int x \sin x \; dx$. **(05)**

OR

(a) Using Vertically and Crosswise Sutra, solve 3251×7604 with proper steps. **(05)**

(b) Find the first and second derivative of the function $x^5 \sin x$ using Urdhva Tiryag sutra and Meru Prastara. **(05)**

Q.2 **(a)** The demand and supply functions for a commodity are $p_d = 56 - x^2$ and $p_s = 8 + \frac{x^2}{3}$. Find the consumer's surplus and producer's surplus at equilibrium price. **(05)**

(b) Find the area of the region bounded by $y^2 = 4x$ and the line $x = 3$. **(05)**

OR

(a) Define: Integration by Parts Rule. Evaluate the following integrals: **(05)**

i. $\int (2 + x) logx \, dx$

ii. $\int \frac{logx}{(x+1)^2} \, dx$

(b) The demand function is $x = \frac{24-2p}{3}$, where x is the number of units demanded and p is the price per unit. Find: **(05)**

i) The revenue function R in terms of p.

ii) The price and the number of units demanded for which the revenue is maximum.

Q.3 **(a)** Find the component form of sum of $\overrightarrow{AB}$ and $\overrightarrow{CD}$, where the vectors are $A = (1, -1), B = (2,0), C = (-1,3)$ and $D = (-2,2)$ **(05)**

(b) If $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$ then find X and Y. **(05)**

OR

(P.T.O)

(a) Let $u = 3i + 2j$, $v = i + 2j$, and $w = 2i - j$. Find scalars a and b such that $u = av + bw$. **(05)**

(b) Find the value of x: $[3 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ **(05)**

Q.4 **(a)** Use the three iterations of the Newton-Raphson method to find the root of the function $f(x) = x^2 - 3$ starting from $x_0 = 2$. **(05)**

(b) Use Descartes's rule of sign and Budan's theorem to find maximum number of positive roots, maximum number of negative roots, and bound of number of real roots of the function $f(x)$ in [0, 2]. Show results in a table. **(05)**

$$f(x) = x^4 - 4x + 1$$

OR

(a) Use the least-square method to fit a straight line to the following data points: $(0,1), (1,2), (2,4)$, and $(3,5)$ **(05)**

(b) Use the Lagrange interpolation approach to find the polynomial that fits the data: $(0,1), (1,3)$, and $(2,5)$ **(05)**

Q.5 Do as directed. Attempt any **TEN** out of **TWELVE:** (Each carries 01 mark) **(10)**

(1) For regression line $Y = 3.76 + 0.4\,X$, what is the estimated value of Y for $X = 10$?

(2) $\frac{d}{dx}(\sin x \cdot \cos x) =$ ______

A. $\cos^2 x + \sin^2 x$ C. $\sin x \cdot \cos x$

B. $-\cos x \cdot \sin x$ D. $\cos^2 x - \sin^2 x$

(3) $\int \frac{(log x)^2}{x} dx =$ _______

A. $\frac{(logx)^2}{2} + c$ C. $\frac{-log}{2} + c$

B. $\frac{logx}{2} + c$ D. $\frac{(logx)^3}{3} + c$

(4) Find adjoint of the matrix A where $A = \begin{bmatrix} -2 & 5 \\ 2 & 3 \end{bmatrix}$

(5) Define: Correlation. State its types.

(6) Define: Average cost and Marginal cost

(7) Find the distance from point $(3, -4, 2)$ to the xy-plane.

(8) If $A = \begin{bmatrix} 2 & 3 & 0 \\ -2 & 2 & 1 \\ 0 & 3 & 2 \end{bmatrix}$ then find transpose of the matrix A.

(9) Find dot product $v.u$ where $v = 2i - 2j + \sqrt{3}k, u = -i + 2j - \sqrt{3}k$.

(10) The Newton's Forward Interpolation is used for the data (0,1), (1,3), and (2,7) then $f(1.5)\ is$:

A. 4
B. 4.25
C. 4.75
D. 5

(11) The maximum number of positive real roots of the polynomial $p(x) = x^4 - 3x^3 + 2x^2 - x + 5$ are:

A. 1
B. 2
C. 3
D. 4

(12) The true value of a measurement is 12.345 and approximate value is 12.3, then relative percentage error (rounded to 4 decimal places) is:

A. 0.3646 %
B. 0.3456 %
C. 0.3444 %
D. 0.3546 %

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