

B.Sc. Sem.-2 Examination

CC-3 - Paper-103

Statistics

May-2025

Time : 2-30 Hours]

[Max. Marks : 70

- Q. 1 (i) Define terms: Sample space, Events, null event, complementary events, Mutually exclusive and exhaustive events. Also, in usual notations, Prove $P(\bar{A}) = 1 - P(A)$ 07
- (ii) In usual notations, state and prove addition rule of probability. Given $P(\bar{A}) = 1/2$, $P(B) = 1/3$, and $P(\bar{A} \cap B) = 1/6$, then, obtain probabilities of $P(A \cup B)$, $P(\bar{A} \cap \bar{B})$ and $P(A \cap \bar{B})$. 07

OR

- Q. 1 (i) Define conditional probability, independence of events. In usual notations, prove that if A and B are independent events, then \bar{A} and B are also independent events. 07
- (ii) State and prove theorem on total probability. Hence or otherwise, prove Bayes' Rule of probability 07
- Q. 2 (i) (i) Random variable, (ii) probability mass function, (iii) probability density function (iv) Probability distribution function. For a random variable X, probability density function is given as under:
 $f(x) = c(x+1)$, $2 < X < 3$
 $= 0$ otherwise 07

- (ii) Determine (i) constant c, (ii) $P(2 < X < 2.5)$
 Define mathematical expectation of a random variable. If a r.v X has a probability distribution as shown in the following table: 07

| | | | | |
|------|---|----|----|----|
| x | 1 | 2 | 3 | 4 |
| p(x) | k | 2k | 2k | 3k |

Determine (i) k, (ii) $E(X)$, $E(3X+5)$, (iii) $V(X)$, $V(2X)$

OR

- Q. 2 (i) Define terms: raw moments, moment generating function, cumulant generating function, skewness and kurtosis. If X follows a probability density function 07

$$f(x) = \begin{cases} \frac{1}{3}, & 1 < X < 4 \\ 0 & \text{otherwise} \end{cases}$$

then find first four raw moments. Also, find the value of μ_2 .

- (ii) For the continuous distribution of a random variable X with the probability density function 07

$$f(x) = \begin{cases} 3x^2 & 0 \leq X \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find constants a and b such that i) $P(X \leq a) = P(X > a)$ and ii) $P(X > b) = 0.5$

- Q. 3 (i) In usual notations, state and prove Boole's inequality. 07
- (ii) Define Markov inequality. State its importance. 07
 Let X has a particular distribution with parameters n and p, with $E(X) = np$. Using Markov's inequality, evaluate an upper bound on $P(X \geq \alpha n)$, for $p = 1/3$ and $\alpha = 3/4$. (where $p < \alpha < 1$)

OR

- Q. 3 (i) Define concave and convex functions. In usual notations, state and prove Jensen's inequality. 07
- (ii) In usual notations, prove Bonferroni inequality. 07

- Q. 4** (i) Define Following terms: **07**
 Two dimensional random variables, marginal distribution and conditional distribution.
 For a random variable X, p.m.f is given as under:

$$P(x) = c(x+1)(y+1) \quad , x = 0, 1, ; y = 1, 2$$

$$= 0 \quad \text{otherwise}$$
 (c is constant to be determined.). Find conditional function of Y given X.
- (ii) Define conditional expectation. **07**
 Show that $E[E(X/Y)] = E(X)$
- OR
- Q. 4** (i) Define: Independence of random variables, product moment, **07**
 The joint p.d.f of r.vs (X,Y) is $f(x,y) = \begin{cases} kxy, & 0 < x < 2, 0 < y < 2 \\ 0 & , \text{otherwise} \end{cases}$
- Check whether the random variables are independent or not.
- (ii) The joint p.d.f. of random variables (X,Y) is **07**

$$f(x,y) = \begin{cases} x+y & , 0 < x < 1, 0 < y < 1 \\ 0 & , \text{elsewhere} \end{cases}$$
 then derive (i) marginal density of X, conditional density of X/ Y, (ii) Joint distribution function.
- Q. 5** Attempt ANY SEVEN (07) **14**
- (i) Define elementary event.
 (ii) What is objective and subjective probability?
 (iii) Define axiomatic definition of probability.
 (iv) Define factorial moments.
 (v) State one use of skewness.
 (vi) What are equi probable elementary events? Give one example.
 (vii) If A, B and C are independent, then are events A and $B \cup C$ also independent?
 (VIII) State types of random variables.
 (ix) Define probability generating function.
